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Fixed Point Theorems for Mapping Having The Mixed Monotone Property

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ABSTRACT

By using the mapping having the mixed monotone property, we have proved a tripled fixed point theorem in partially ordered metric space.
Keywords: *Partially ordered set, complete metric space, tripled fixed point, mixed monotone property.*

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INTRODUCTION AND PRELIMINARIES

Ran and Reurings [1] firstly discussed the existence and uniqueness of fixed point for contraction type mappings in 2004. Agarwal *et al* [5], Bhaskar and Lakshmikantham [6], Lakshmikantham and Ciric [8], Nieto and Lopez [3], and Berinde and Borcut [7] proved some famous and well known results for the existence of a fixed point in partially ordered metric space. In 1987,Guo and Lakshmikantham[2] reported with the notion of coupled fixed point. Bhaskar and Lakshmikantham [6] reconsidered the concept of coupled fixed point in partially ordered metric space in 2006.The notion of tripled fixed point was introduced by Berinde and Borcut [7] They proved some tripled and n-tupled fixed point theorems and discussed the existence and uniqueness of solutions under different conditions.

In this paper, we have derived a new tripled fixed point theorem for mapping having the mixed monotone property in partially ordered metric space .

Definition 1.1. A partially ordered set is a set X with a binary operation ≤ denoted by $(X, \leq)$ such that for all $p, q, r \in X$

(i) $p \leq p$ (reflexivity)

(ii) $p \leq q$ and $q \leq p \Rightarrow p = q$ (anti-symmetry)

(iii) $p \leq q$ *and* $q \leq r \Rightarrow p \leq r$ (transitivity).

Definition 1.2. A sequence (x_n) in a metric space (X, d) is said to converge to a point $x \in X$ denoted by $\lim_{n\to\infty} x_n = x$ if $\lim_{n\to\infty} d(x_n, x) = 0$.

Definition 1.3. A sequence (x_n) in a metric space (X, d) is said to be Cauchy Sequence

if $\lim_{t\to\infty} d(x_{n,} x_m) = 0$ *for all* $n, m > t$.

Definition 1.4. A metric space (X, d) is complete if every Cauchy sequence in X is convergent.

Definition 1.5. [7] Let X be a non-empty set and $F: X^3 \to X$ be a map. An element $(x, y, z) \in X^3$ is called a tripled fixed point of F if $F(x, y, z) = x$, $F(y, x, y) = y$, $F(z, y, x) = z$.

Definition 1.6. [7] Let$(X, \leq)$be a partially ordered set and $F: X^3 \to X$. The mapping F is said to have mixed monotone property if $F(x, y, z)$ is monotone non-decreasing in x and z and is monotone non-increasing in y that is for $x, y, z \in X$,

$x_{1,}x_2 \in X, x_1 \leq x_2 \Rightarrow F(x_1, y, z) \leq F(x_2, y, z),$

$y_{1,}y_2 \in X, y_1 \leq y_2 \Rightarrow F(x, y_1, z) \geq F(x, y_2, z),$

$z_{1,}z_2 \in X, z_1 \leq z_2 \Rightarrow F(x, y, z_1) \leq F(x, y, z_2).$

MAIN RESULT

Theorem 2.1. Let(X, ≤) be a partially ordered complete metric space. Let $F: X^3 \to X$ be a continuous mapping having the mixed monotone property on X. Assume that there exists a $\beta \in [0,1)$ with $d\big(F(x, y, z), F(u, v, w)\big)$

$\leq \beta \max\left\{\frac{d(x,F(x,y,z))d(u,F(u,v,w))}{d(x,u)}, \frac{d(u,F(x,y,z))d(x,F(u,v,w))}{d(x,u)}, d(x,u)\right\}$ (2.1.1)

for all $x \geq u, y \leq v$ and $z \geq w$,

and if there exist points $x_0, y_0, z_0 \in X$ with $x_0 \leq F(x_0, y_0, z_0),\ y_0 \geq F(y_0, x_0, y_0), z_0 \leq F(z_0, y_0, x_0)$,

then F has a tripled fixed point in X^3.

Remark: If we have $F: X^2 \to X$ then our theorem reduces to theorem (3.1) of Ramakant Bhardwaj [4].

Proof: Let $x_0, y_0, z_0 \in X$ with

$x_0 \leq F(x_0, y_0, z_0), y_0 \geq F(y_0, x_0, y_0), z_0 \leq F(z_0, y_0, x_0).$ (2.1.2)

Define the sequence $(x_n), (y_n)$ and (z_n) in X such that

$x_{n+1} = F(x_n, y_n, z_n)$

$y_{n+1} = F(y_n, x_n, y_n)$

$z_{n+1} = F(z_n, y_n, x_n)$ *for all* $n = 0,1,2 \cdots$ (2.1.3)

We claim that $(x_n), (z_n)$ are non- decreasing and (y_n) is non-increasing , that is,

$x_n \leq x_{n+1},\ y_n \geq y_{n+1},\ z_n \leq z_{n+1}$. (2.1.4)

From (2.1.2) and (2.1.3), we have

$x_0 \leq F(x_0, y_0, z_0),\ y_0 \geq F(y_0, x_0, y_0),\ z_0 \leq F(z_0, y_0, x_0)$,

$x_1 = F(x_0, y_0, z_0),\ y_1 = F(y_0, x_0, y_0),\ z_1 = F(z_0, y_0, x_0)$.

$\Rightarrow\ x_0 \leq x_1,\ y_0 \geq y_1,\ z_0 \leq z_1$.

That is equation (2.1.4) holds for n=0.

Now suppose that equation (2.1.4) holds for some n, that is,

$x_n \leq x_{n+1},\ y_n \geq y_{n+1},\ z_n \leq z_{n+1}$.

We shall prove that equation (2.1.4) is true for n+1.

Now $x_n \leq x_{n+1},\ y_n \geq y_{n+1},\ z_n \leq z_{n+1}$.

Then by mixed monotone property of F, we have

$$
\begin{aligned}
x_{n+2} &= F(x_{n+1}, y_{n+1}, z_{n+1}) \geq F(x_n, y_{n+1}, z_{n+1}) \\
&\geq F(x_n, y_n, z_{n+1}) \geq F(x_n, y_n, z_n) = x_{n+1}, \\
y_{n+2} &= F(y_{n+1}, x_{n+1}, y_{n+1}) \leq F(y_n, x_{n+1}, y_{n+1}) \\
&\leq F(y_n, x_n, y_{n+1}) \leq F(y_n, x_n, y_n) = y_{n+1}, \\
z_{n+2} &= F(z_{n+1}, y_{n+1}, x_{n+1}) \geq F(z_n, y_{n+1}, x_{n+1}) \\
&\geq F(z_n, y_n, x_{n+1}) \geq F(z_n, y_n, x_n) = z_{n+1}.
\end{aligned}
$$

Thus by mathematical induction principle, equation (2.1.4) holds for all $n \in N$.

So $x_0 \leq x_1 \leq x_2 \leq \cdots \leq x_n \leq x_{n+1} \cdots$

$y_0 \geq y_1 \geq y_2 \geq \cdots \geq y_n \geq y_{n+1} \cdots$

$z_0 \leq z_1 \leq z_2 \leq \cdots \leq z_n \leq z_{n+1} \cdots$

Now as $x_n \geq x_{n-1}, y_n \leq y_{n-1}, z_n \geq z_{n-1}$ so from (2.1.1), we have

$$
d\big(F(x_n, y_n, z_n), F(x_{n-1}, y_{n-1}, z_{n-1})\big) \leq \beta \max\left\{\frac{d(x_n, F(x_n, y_n, z_n))d(x_{n-1}, F(x_{n-1}, y_{n-1}, z_{n-1}))}{d(x_n, x_{n-1})}, \frac{d(x_{n-1}, F(x_n, y_n, z_n))d(x_n, F(x_{n-1}, y_{n-1}, z_{n-1})}{d(x_n, x_{n-1})}, d(x_n, x_{n-1})\right\}
$$

$\Rightarrow d(x_{n+1}, x_n) \leq \beta \max\{d(x_n, x_{n+1}), 0, d(x_n, x_{n-1})\}$

If we take $\max\{d(x_n, x_{n+1}), 0, d(x_n, x_{n-1})\}$ equal to $d(x_{n+1}, x_n)$,

then $d(x_{n+1}, x_n) \leq \beta\, d(x_{n+1}, x_n)$, which is a contradiction to the hypothesis.

$\Rightarrow d(x_n, x_{n+1}) \leq \beta\, d(x_n, x_{n-1})$. $\cdots$ (2.1.5)

Again, since $x_n \geq x_{n-1}, y_n \leq y_{n-1}$ so from(2.1.1), we have

$$
d\big(F(y_{n-1}, x_{n-1}, y_{n-1}), F(y_n, x_n, y_n)\big) \leq \beta \max\left\{\frac{d(y_{n-1}, F(y_{n-1}, x_{n-1}, y_{n-1}))d(y_n, F(y_n, x_n, y_n))}{d(y_n, y_{n-1})}, \frac{d(y_n, F(y_{n-1}, x_{n-1}, y_{n-1}))d(y_{n-1}, F(y_n, x_n, y_n)}{d(y_n, y_{n-1})}, d(y_n, y_{n-1})\right\}
$$

$\Rightarrow d(y_{n+1}, y_n) \leq \beta \max\{d(y_{n+1}, y_n), 0, d(y_n, y_{n-1})\}$.

If we take $\max\{d(y_{n+1}, y_n), 0, d(y_n, y_{n-1})\}$ equal to $d(y_{n+1}, y_n)$,

then $d(y_{n+1}, y_n) \leq \beta\, d(y_{n+1}, y_n)$, which is again a contradiction to the hypothesis.

$\Rightarrow d(y_n, y_{n+1}) \leq \beta\, d(y_n, y_{n-1})$. (2.1.6)

Similarly $d(z_n, z_{n+1}) \leq \beta\, d(z_n, z_{n-1})$. (2.1.7)

Adding (2.1.5), (2.1.6) and (2.1.7), we get

$d(x_{n+1}, x_n) + d(y_{n+1}, y_n) + d(z_{n+1}, z_n)$

$\leq \beta\{d(x_n, x_{n-1}) + d(y_n, y_{n-1}) + d(z_n, z_{n-1})\}$ *where* $\beta < 1$. (2.1.8)
Let us denote the left hand side of (2.1.8) by d_n and use similar notation for right hand side of (2.1.8).
then $d_n \leq \beta d_{n-1}$.
Similarly we can derive $d_{n-1} \leq \beta d_{n-2}$ and so on.
We get $d_n \leq \beta d_{n-1} \leq \beta^2 d_{n-2} \leq \cdots \leq \beta^n d_0$. (2.1.9)

$\Rightarrow \lim_{n\to\infty} d_n = \lim_{n\to\infty}\{d(x_{n+1}, x_n) + d(y_{n+1}, y_n) + d(z_{n+1}, z_n)\} = 0.$

$\Rightarrow \lim_{n\to\infty} d(x_{n+1}, x_n) = \lim_{n\to\infty} d(y_{n+1}, y_n) = \lim_{n\to\infty} d(z_{n+1}, z_n) = 0$.

For each $m \geq n$, *we have*

$d(x_m, x_n) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{m-1}, x_m),$

$d(y_m, y_n) \leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \cdots + d(y_{m-1}, y_m),$

$d(z_m, z_n) \leq d(z_n, z_{n+1}) + d(z_{n+1}, z_{n+2}) + \cdots + d(z_{m-1}, z_m).$

By adding we get,

$$d(x_m, x_n) + d(y_m, y_n) + d(z_m, z_n) \leq d_n + d_{n+1} + \cdots d_{m-1} \leq (\beta^n + \beta^{n+1} + \cdots + \beta^{m-1})d_0 \leq \frac{\beta^n}{1-\beta} d_0.$$

$\lim_{n\to\infty}\{d(x_m, x_n) + d(y_m, y_n) + d(z_m, z_n)\} = 0.$

Hence $(x_n), (y_n), (z_n)$ are Cauchy sequences in X.
Since X is a complete metric space, there exists x, y, z ∈ X such that
$\lim_{n\to\infty} x_n = x, \quad \lim_{n\to\infty} y_n = y, \quad \lim_{n\to\infty} z_n = z$.
Thus by taking limits as $n\to\infty$ in equation (2.1.3) we get

$x = \lim_{n\to\infty} x_n = \lim_{n\to\infty} F(x_{n-1}, y_{n-1}, z_{n-1})$
$= F(\lim_{n\to\infty} x_{n-1}, \lim_{n\to\infty} y_{n-1}, \lim_{n\to\infty} z_{n-1}) = F(x, y, z),$

$y = \lim_{n\to\infty} y_n = \lim_{n\to\infty} F(y_{n-1}, x_{n-1}, y_{n-1})$
$= F(\lim_{n\to\infty} y_{n-1}, \lim_{n\to\infty} x_{n-1}, \lim_{n\to\infty} y_{n-1}) = F(y, x, y),$

$z = \lim_{n\to\infty} z_n = \lim_{n\to\infty} F(z_{n-1}, y_{n-1}, x_{n-1})$
$= F(\lim_{n\to\infty} z_{n-1}, \lim_{n\to\infty} y_{n-1}, \lim_{n\to\infty} x_{n-1}) = F(z, y, x).$

Hence $F(x, y, z) = x, \ F(y, x, y) = y, \ F(z, y, x) = z$.
Hence F has a tripled fixed point.

Theorem 2.2. Let(X, d, ≤) be a partially ordered complete metric space. Let $F: X^3 \to X$ be a continuous mapping having the mixed monotone property on X. Assume that there exists a $\beta \in [0,1)$ with
$d\big(F(x, y, z), F(u, v, w)\big) \leq \beta \max\{d(u, F(x, y, z), d(x, F(u, v, w)\}$
for all $x \geq u, y \leq v$ and $z \geq w$ (*)
and if there exist points $x_0, y_0, z_0 \in X$ with $x_0 \leq F(x_0, y_0, z_0), \ y_0 \geq F(y_0, x_0, y_0), z_0 \leq F(z_0, y_0, x_0)$
then F has a tripled fixed point in X^3.

Remark: If we have$F: X^2 \to X$ then our theorem reduces to theorem (3.2) of Ramakant Bhardwaj [4].

Proof. Let $x_0, y_0, z_0 \in X$ with
$x_0 \leq F(x_0, y_0, z_0), y_0 \geq F(y_0, x_0, y_0), z_0 \leq F(z_0, y_0, x_0)$ (2.2.1)
*D*efine the sequence $(x_n), (y_n)$ and (z_n) in X such that
$x_{n+1} = F(x_n, y_n, z_n),$
$y_{n+1} = F(y_n, x_n, y_n),$
$z_{n+1} = F(z_n, y_n, x_n)$ *for all* $n = 0,1,2 \cdots$ (2.2.2)
We claim that $(x_n), (z_n)$ are non- decreasing and (y_n) is non-increasing , that is,
$x_n \leq x_{n+1}, \ y_n \geq y_{n+1}, \ z_n \leq z_{n+1}$. (2.2.3)
From (2.2.1) and (2.2.2), we have
$x_0 \leq F(x_0, y_0, z_0), \ y_0 \geq F(y_0, x_0, y_0), \ z_0 \leq F(z_0, y_0, x_0),$
$x_1 = F(x_0, y_0, z_0), \ y_1 = F(y_0, x_0, y_0), \ z_1 = F(z_0, y_0, x_0).$

$\Rightarrow \quad x_0 \leq x_1, \ y_0 \geq y_1, \ z_0 \leq z_1$.
That is equation (2.2.3) holds for n=0.
Now suppose that equation (2.2.3) holds for some n, that is
$x_n \leq x_{n+1}, \ y_n \geq y_{n+1}, \ z_n \leq z_{n+1}$.
We shall prove that equation (2.2.3) is true for n+1
Now $x_n \leq x_{n+1}, \ y_n \geq y_{n+1}, \ z_n \leq z_{n+1}$.
Then by mixed monotone property of F, we have
$x_{n+2} = F(x_{n+1}, y_{n+1}, z_{n+1}) \geq F(x_n, y_{n+1}, z_{n+1})$
$\geq F(x_n, y_n, z_{n+1}) \geq F(x_n, y_n, z_n) = x_{n+1},$
$y_{n+2} = F(y_{n+1}, x_{n+1}, y_{n+1}) \leq F(y_n, x_{n+1}, y_{n+1})$

$\le F(y_n, x_n, y_{n+1}) \le F(y_n, x_n, y_n) = y_{n+1}$,
$z_{n+2} = F(z_{n+1}, y_{n+1}, x_{n+1}) \ge F(z_n, y_{n+1}, x_{n+1})$
$\ge F(z_n, y_n, x_{n+1}) \ge F(z_n, y_n, x_n) = z_{n+1}$.
Thus by mathematical induction principle equation (2.2.3) holds for all $n \in N$
So $x_0 \le x_1 \le x_2 \le \cdots \le x_n \le x_{n+1} \cdots$
$y_0 \ge y_1 \ge y_2 \ge \cdots \ge y_n \ge y_{n+1} \cdots$
$z_0 \le z_1 \le z_2 \le \cdots \le z_n \le z_{n+1} \cdots$
Now $x_n \ge x_{n-1}, y_n \le y_{n-1}, z_n \ge z_{n-1}$ so from $(*)$, we have

$d(F(x_n, y_n, z_n), F(x_{n-1}, y_{n-1}, z_{n-1}) \le \beta \max\{d(x_{n-1}, F(x_n, y_n, z_n)), d(x_n, F(x_{n-1}, y_{n-1}, z_{n-1}))\}$,
$d(x_{n+1}, x_n) \le \beta \, max \{d(x_{n-1}, x_{n+1}), 0\}$.
This implies, $d(x_{n+1}, x_n) \le \frac{\beta}{1-\beta} \, d(x_n, x_{n-1})$ (2.2.4)
Now $x_n \ge x_{n-1}, y_n \le y_{n-1}$ so from (*), we have,
$d(F(y_{n-1}, x_{n-1}, y_{n-1}), F(y_n, x_n, y_n) \le \beta \max\{d(y_n, F(y_{n-1}, x_{n-1}, y_{n-1})), d(y_{n-1}, F(y_n, x_n, y_n))\}$,
$d(y_{n+1}, y_n) \le \beta \, max \{0, d(y_{n-1}, y_{n+1})\}$.
This implies, $d(y_{n+1}, y_n) \le \frac{\beta}{1-\beta} \, d(y_n, y_{n-1})$ (2.2.5)
Similarly, since $z_{n-1} \le z_n, y_{n-1} \ge y_n, x_{n-1} \le x_n$,
$d(z_{n+1}, z_n) \le \frac{\beta}{1-\beta} \, d(z_n, z_{n-1})$. (2.2.6)

Adding(2.2.4), (2.2.5) and (2.2.6) ,we get,

$d(x_{n+1}, x_n) + d(y_{n+1}, y_n) + d(z_{n+1}, z_n)$
$\le h\{d(x_n, x_{n-1}) + d(y_n, y_{n-1}) + d(z_n, z_{n-1})\} \; where \; h = \frac{\beta}{1-\beta} < 1$. (2.2.7)
Let $d(x_{n+1}, x_n) + d(y_{n+1}, y_n) + d(z_{n+1}, z_n) = \alpha_n$ then (2.2.7) becomes
$\alpha_n \le h\alpha_{n-1}$.
Similarly we can derive $\alpha_{n-1} \le h\alpha_{n-2}$ and so on.
We get $\alpha_n \le h\alpha_{n-1} \le h^2\alpha_{n-2} \le \cdots \le h^n\alpha_0$. (2.2.8)
$\Rightarrow \lim_{n\to\infty} \alpha_n = \lim_{n\to\infty}\{d(x_{n+1}, x_n) + d(y_{n+1}, y_n) + d(z_{n+1}, z_n)\} = 0$.
$\Rightarrow \lim_{n\to\infty} d(x_{n+1}, x_n) = \lim_{n\to\infty} d(y_{n+1}, y_n) = \lim_{n\to\infty} d(z_{n+1}, z_n) = 0$.
For each $m \ge n$, *we have*
$d(x_m, x_n) \le d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{m-1}, x_m)$,
$d(y_m, y_n) \le d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \cdots + d(y_{m-1}, y_m)$,
$d(z_m, z_n) \le d(z_n, z_{n+1}) + d(z_{n+1}, z_{n+2}) + \cdots + d(z_{m-1}, z_m)$.
By adding we get
$d(x_m, x_n) + d(y_m, y_n) + d(z_m, z_n) \le \alpha_n + \alpha_{n+1} + \cdots \alpha_{m-1} \le (h^n + h^{n+1} + \cdots + h^{m-1})\alpha_0$
$\le \frac{h^n}{1-h}\alpha_0$.
$\lim_{n\to\infty}\{d(x_m, x_n) + d(y_m, y_n) + d(z_m, z_n)\} = 0$
Hence $(x_n), (y_n), (z_n)$ are Cauchy sequences in X.
Since X is a complete metric space, there exists x, y. , z ∈ X such that
$\lim_{n\to\infty} x_n = x, \quad \lim_{n\to\infty} y_n = y, \quad \lim_{n\to\infty} z_n = z$.
Thus by taking limits as $n\to\infty$ in equations (2.2.2) we get
$x = \lim_{n\to\infty} x_n = \lim_{n\to\infty} F(x_{n-1}, y_{n-1}, z_{n-1})$
$= F(\lim_{n\to\infty} x_{n-1}, \lim_{n\to\infty} y_{n-1}, \lim_{n\to\infty} z_{n-1}) = F(x, y, z)$,
$y = \lim_{n\to\infty} y_n = \lim_{n\to\infty} F(y_{n-1}, x_{n-1}, y_{n-1})$
$= F(\lim_{n\to\infty} y_{n-1}, \lim_{n\to\infty} x_{n-1}, \lim_{n\to\infty} y_{n-1}) = F(y, x, y)$,
$z = \lim_{n\to\infty} z_n = \lim_{n\to\infty} F(z_{n-1}, y_{n-1}, x_{n-1})$
$= F(\lim_{n\to\infty} z_{n-1}, \lim_{n\to\infty} y_{n-1}, \lim_{n\to\infty} x_{n-1}) = F(z, y, x)$,
Hence $F(x, y, z) = x, \;\; F(y, x, y) = y, \;\; F(z, y, x) = z$.
Hence F has a tripled fixed point.

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