**International Archive of Applied Sciences and Technology**
Int. Arch. App. Sci. Technol; Vol 6 [1] March 2015: 22-25

[ISO9001: 2008 Certified Organization]
www.soeagra.co/iaast.html

IAAST
ONLINE ISSN 2277- 1565
PRINT ISSN 0976 - 4828

**CODEN: IAASCA** **ORIGINAL ARTICLE**

# Helicity-zero Massless Particles and their Analysis

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**ABSTRACT**
*It is well known that helicity-zero state of a massless particle, like photon, do not exist. However, existence of such states cannot be denied on the theoretical ground. In this paper we have tried to propose a theoretical study of massless particle existing in helicity $+1,0$, and $-1$ states. It is a fact that a massless vector field of helicity $\pm 1$ transform inhomogeneously under a general Lorentz transformation. We have shown that the presence of inhomogeneous term in Lorentz transformation rule for this vector field can be very well associated with presence of a helicity-zero massless particle. It has also been shown that the construction of a four-vector like field for helicity $\pm 1$ massless particle could be a viable possibility provided the existence of a helicity-zero four-vector field for a massless particle is presupposed.*
***Keywords*** *Four-vector field, Massless particle, Helicity-zero states, Photon*
PACS number(s): 03.70. + k, 14.70. Bh
**Received 08/01/2015** **Revised 27/02/2015** **Accepted 04/03/2015**

**Citation of this article**
Mahendra J , Sanjay S. Helicity-zero Massless Particles and their Analysis. Int. Arch. App. Sci. Technol; Vol 6 [1] March 2015: 22-25. DOI.10.15515/iaast.0976-4828.6.1.2225

## INTRODUCTION

In the development of the quantum field theory photon (a massless particle with helicity $\pm 1$) has played an important role. In fact quantum field theory of electromagnetism was developed for photon. Topics like quantum entanglement [1], photon polarization [2-3] are based on photons and their properties. With the invent of modern techniques, such as quantum information theory, relativistic quantum information theory [4], the behavior of a quantum system can be described and predicted in an effective and useful way. Comparatively new quantum applications like entanglement-enhanced communication, quantum teleportation, and quantum-enhanced global positioning, involve photons and other quantum particles as quantum information carriers. We notice, however, that the focus has always been on physical photons of helicity one and in some sense these spin-one massless particles really play a fundamental role in our physical life. As the quanta of vector field, particles of spin one are of great importance in quantum field theory and when it comes specifically to the spin, both massive and massless particles require equal attention.

For massive vector particle of spin one there are usual three helicity states. In the zero-mass limit of a massive particle, however, we observe only two possible spin values along the direction of motion. In fact a massless particle like photon has only one helecity state. For parity conservation this particle should be accompanied by another particle of opposite helicity. However, we notice that spin of a particle comes first and then comes the helecity. So if there is a spin-one particle, whether it is massive or massless ($m \to 0$), there should be three helicity states present.

In this paper we have tried to show that within the framework of quantum field theory we cannot avoid simultaneous presence of helicity-zero massless vector particles (helicity-zero photon) along with the usual helicity-one massless particles. Though we do not see a vast literature, covering topics on the properties of these particles, yet different manifestations of these particles appear vividly in different theories [5-6].

## THEORETICAL ANALYSIS

The state $|\Psi_{p,\sigma}\rangle$ for a massless free particle transforms under unitary Lorentz transformation as [7]

$$U(L)|\Psi_{p,\sigma}\rangle = \sqrt{\frac{(Lp)^0}{p^0}}exp\big(i\sigma\theta(L,p)\big)|\Psi_{Lp,\sigma}\rangle \tag{1}$$

where $L$ is homogeneous Lorentz transformation, $\sigma$ is helecity, $\mathbf{p}$ is momentum of the particle and $\theta$ is wigner angle [8]. Construction of a free field for a massless particle requires Lorentz invariance of S-matrix [7]. To maintain Lorentz covariance of S-matrix, creation and annihilation fields have been introduced.

It is the principal of locality which demands the quantum fields to commute for space-like separations. Therefore these fields are constructed using creation and annihilation operators. A linear combination of the creation operator for antiparticle and corresponding annihilation operator for particle of momentum $\mathbf{p}$ and helecity $\sigma$, can be used to get a general expression of free field for a massless particle.
For homogeneous Lorentz transformation these operators (creation operator $a^\dagger$ and annihilation operator $a$) transform as

$$U(L)a^\dagger(\mathbf{p},\sigma)U^{-1}(L) = \sqrt{\frac{(Lp)^0}{p^0}}exp\big(i\sigma\theta(L,p)\big)a^\dagger(\mathbf{p}_L,\sigma) \tag{2}$$

$$U(L)a(\mathbf{p},\sigma)U^{-1}(L) = \sqrt{\frac{(Lp)^0}{p^0}}exp\big(-i\sigma\theta(L,p)\big)a(\mathbf{p}_L,\sigma) \tag{3}$$

Since we are considering massless particle case we have $|\mathbf{p}| = p^0$. In fact for massless case the standard four-momentum [7] is $k^\mu = (0,0,k,k)$, so we get $|\mathbf{p}| = p^0$ when $k^\mu$ changes to $p^\mu$ under a Lorentz transformation. Now for a particle of given spin the quantum field theory develops around the finite-dimensional and non-unitary irreducible representations of the poincaré symmetry. This is just opposite to the nature of representations of poincaré symmetry on the Hilbert space, as representations are infinite-dimensional and unitary here. This sort of dissimilarity between the representations on the fields and the Hilbert space requires introduction of gauge symmetry. To configure a field which transforms in the irreducible representations of the Lorentz group one must work in configuration space. Therefore while constructing expression for a casual field, creation and annihilation operators are integrated over all spatial momenta. Now, the general form of a casual field that transform according to the general irreducible representation $(A,B)$ of the homogeneous Lorentz group [7] can be written as

$$\phi^{ab}(x) = (2\pi)^{-3/2}\sum_\sigma\int(2p_0)^{-1/2}\,d^3p[a(\mathbf{p},\sigma)e^{ip.x}u^{ab}(\mathbf{p},\sigma) + a^\dagger(\mathbf{p},\sigma)e^{-ip.x}v^{ab}(\mathbf{p},\sigma)] \tag{4}$$

Where $a$ and $b$ are labels of irreducible representations. It has been shown [7] that a massless field of type $(A,B)$ can be constructed from the creation operator for antiparticle of helecity $-\sigma$, and annihilation operator for particle of helicity $\sigma$. Here $\sigma$ is given as

$$\sigma = B - A \tag{5}$$

In an effort to construct a four-vector $\left[\left(\frac{1}{2},\frac{1}{2}\right)\right]$ field for massless, helecity $\pm1$ particle, we come across a situation that does not fulfill our requirement of getting a general four-vector field [7]. However, one can choose an expression of casual vector field to start with, and hence for $\left[\left(\frac{1}{2},\frac{1}{2}\right)\right]$ representation the required four-vector like field expression would be

$$A^\mu(x) = (2\pi)^{-3/2}\sum_\sigma\int(2p_0)^{-1/2}\,d^3p[a(\mathbf{p},\sigma)e^{ip.x}\,u^\mu(\mathbf{p},\sigma) + a^\dagger(\mathbf{p},\sigma)e^{-ip.x}v^\mu(\mathbf{p},\sigma)] \tag{6}$$

Where index $\mu$ runs over the four spacetime coordinate labels 1, 2, 3 and 0, and $\sigma = \pm1$. Lorentz transformation properties of this field can be obtained using (2) and (3) as

$$U(L)A^\mu(x)U^{-1}(L) = (2\pi)^{-3/2}\int\frac{d^3(Lp)}{\sqrt{2(Lp)^0}}\begin{bmatrix} exp\big(-i\sigma\theta(L,p)\big)a(\mathbf{p}_L,\sigma)e^{ip.x}u^\mu(\mathbf{p},\sigma) \\ +exp\big(i\sigma\theta(L,p)\big)\,a^\dagger(\mathbf{p}_L,\sigma)e^{-ip.x}v^\mu(\mathbf{p},\sigma)\end{bmatrix} \tag{7}$$

It can be shown [9] that for a general Lorentz transformation, (7) can be written as

$$U(L)A^\mu(x)U^{-1}(L) = (L^{-1})^\mu{}_\nu A^\nu(Lx) + i\frac{\partial}{\partial\,(Lx)_\mu}\Omega(L,x) \tag{8}$$

where $\Omega(L,x)$ is linear in particle annihilation and antiparticle creation operators. Thus we observe that

$A^{\mu}(x)$ does not quite behave like a four-vector. Moreover, the Lorentz transformation rule for $\Omega(L,x)$ can be written as

$$U(\acute{L})\Omega(L,x)U^{-1}(\acute{L}) = \Omega(\acute{L}L,x) - \Omega(\acute{L},Lx) \qquad (9)$$

Which shows its non-scalar nature. It has been shown [7] that $A^{\mu}(x)$ can be a part of Lorentz-invariant physical theories provided there exists a conserved four-current $J_{\mu}$. Another requirement is that as a coupling of $A^{\mu}$, this four-current should be invariant under a specific gauge transformation.

Now what follows (9) clearly shows that we have deliberately and cleverly avoided the in homogenety in (8). The very idea behind this simplification of (8) can be exploited to get some interesting results we are trying to put forward. We note that in (8) $A^{\mu}(x)$ behaves as a four-vector only if the extra inhomogeneous term does not have an effect. This immediately shows that there is a possibility to construct a four-vector field as a linear combination of the creation and annihilation operators for helicity ±1. However, it has been shown [7] that it is not possible to construct a massless four-vector field with $\sigma = \pm 1$, which transforms homogeneously under Lorentz transformation. This contradicts our claim.

We, however, can sort it out by claiming that by introducing $J_{\mu}$ we have, in fact, provided an alternative way to construct a four-vector field for massless helicity $\pm 1$ particle case. This alternative approach simultaneously demands the following condition to fulfill:

$$\frac{\partial}{\partial\,(Lx)_{\mu}}\Omega(L,x) = 0 \qquad (10)$$

This eq. is somewhat similar to the Lorentz gauge condition [10]. Now we have already shown that this field $\Omega(L,x)$ is of non-scalar nature. Obviously $\Omega(L,x)$ behaves like a four-vector field. Note that here we are treating Lorentz gauge condition as a subsidiary condition, not to eliminate the superfluous time-like and longitudinal part of $\Omega(L,x)$ but to show that the quantity in front of $\frac{\partial}{\partial\,(Lx)_{\mu}}$ should be a four-vector. We also notice that (10) is a consequence of a gauge transformation

$$A^{\mu} \rightarrow A^{\mu} + \partial^{\mu}\Omega \qquad (11)$$

Under which the couplings of $A^{\mu}$, as mentioned previously, remains invariant. The role of (11) is somewhat important in our case. We note that in the theory of Higg's mechanism [5] introduction of a gauge transformation is meant for disappearance of a massless boson (Goldstone boson) of zero spin. In this way the gauge field acquires mass. But here in our case we do not demand disappearance of Ω field, and simply assume its presence in the form of a function of spacetime. The precise form of this field can be written as

$$\Omega(L,x) = (2\pi)^{-3/2}\int (2p^0)^{-1/2}\, d^3p\, \frac{(L^{-1})^0{}_{\nu}}{(L^{-1}p)^0}\left[a(\mathbf{p},\sigma)e^{ip.Lx}u^{\nu}(\mathbf{p},\sigma) - a^{\dagger}(\mathbf{p},\sigma)e^{-ip.Lx}u^{*\nu}(\mathbf{p},\sigma)\right] \qquad (12)$$

Now if it is a vector field it must transform according to $\left[\left(\frac{1}{2},\frac{1}{2}\right)\right]$ representation and hence should represent zero-helecity massless particle state. Here we note that expression (12), just like (6), is also a combination of creation and annihilation operators yet these two expression are different altogether in the sense that (6) is not a four-vector until $\Omega(L,x)$ is proved to be equal to a four-vector.

It is well known that a massive spin one particle can have three spin states namely helicity $+1$,0, and $-1$ states. It is expected that in the limit $m \rightarrow 0$ all these states should also be present. This conjecture gets support from what we have tried to propose in this paper, as (6) along with (12) shows that all three helicity states, for massless particle, are simultaneously present. Hence we can firmely say that as elements of a vector space the zero helicity states are there, but it appears that under quantum mechanical construction and interpretation they concide with the zero vector and therefore are unphysical and can not be observed as real particle. But for non-availability of zero helicity state for massless particle (like photon) , we could not deny the possibility of these states being unstable, after all it's a massless boson of zero spin whose absence justifies presence of electroweak boson.

## CONCLUSION

We have shown that just like the massive particle of spin one, a massless particle (like photon) can also exist in three spin states namely helicity $+1$,0, and $-1$ states. In fact we notice that the concept of gauge invariance and hence the introduction of a conserved four-current itself requires the presence of helicity-zero photons while constructing a four-vector field theory for physical photons of helicity one. We also observe that in our case the Lorentz gauge condition does not exclude the possibility for a helicity-zero four-vector massless particle to exist.

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