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The Study of Lawson Criterion in Fusion Systems for the Reactions ${}^{3}He\,(D,P)\alpha$ and ${}^{11}B\,(p,\alpha)2\alpha$

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ABSTRACT
We have obtained the parameter of the fusion reaction rate (Sigmavi)for the important fusion reactions ${}^{3}He(D, p)\alpha$ and ${}^{11}B(p,\alpha)2\alpha$.Using the result, we have obtained energy balance equation which is the formula of Lawson criterion.
Keywords:*Parameter of the Fusion Reaction Rate (Sigmavi)*, *Lawson Criterion*, *Fusion Reactions*, *Bermesstrahlung radiations*
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INTRODUCTION

In fusion systems, some energy is lost by radiation or other processes. In this article we consider energy losses through Bermesstrahlung radiations for ${}^{3}He(D, p)\alpha$ and ${}^{11}B(p,\alpha)2\alpha$reactions. In fact, external energy is injected to the fusion system to compensate the lost energy. Consequently, we obtain output energy from the fusion reaction. From practical point of view, the energy obtained must be more than the input energy or the source of external energy $(E_{out} > E_{in})$ [1,2]. This balance equation of energy is the Lawson criterion.

SUMMARY OF THE THEORY

Generally, if two cores **a** and **b** do fusion reaction according to the following relation, they produce **e** and **d**.

$$a + b \rightarrow d + e + Q_{ab} \tag{1}$$

Q_{ab} Is the energy released from the fusionreaction. The power of fusion reaction is expressed by the following relation [1].

$$P_{fu} = N_a N_b < \sigma V >_{ab} Q_{ab} \tag{2}$$

N_a and N_b are densities of core **a** and **b** respectively. The quantity $N_a N_b < \sigma V >_{ab}$ is the fusion reaction rate. The quantity $< \sigma V >_{ab}$ is the parameter of fusion reaction rate that is called Sigmavi and shown as follows [1-3].

$$< \sigma V >_{ab} = \int_{V_a}\int_{V_b} f_a(v_a) f_b(v_b) \sigma_{ab}\left(\left|\vec{v}_a - \vec{v}_b\right|\right)\left|\vec{v}_a - \vec{v}_b\right| d^3v_a d^3v_b \tag{3}$$

σ_{ab} is relative cross-section of the reaction, and is measured in Barn. Trough fitting $\sigma(E)=\frac{S(E)}{E}P(E)$ model [4,5] by experimental data, cross-section can be approximately obtained. Here, the structure factor S(E) depends
weakly on energy E or is even assumed to be a constant. The main energy dependence comes from the potential barrier PenetrabilityP(E). It is defined by the ratio of the particle flux traversing the respective potential barrier to the incident flux.The functions $f_a(v_a)$ and $f_b(v_b)$ are the Maxwell speed distribution functions[1-3]. by using $f_a(v_a)$, $f_b(v_b)$ and $\sigma(E)$ in eq.3 we have [3,5] :

$$<\sigma V>_{ab}=\left(\frac{8}{\mu\pi}\right)^{\frac{1}{2}}(kT)^{\frac{-3}{2}}\int_0^\infty E\sigma(E)e^{\frac{-E}{kT}}dE \qquad (4)$$

Since the cross-section function $\sigma(E)$ and the integral of eq.4 are computed approximately, so there is often a high rate of error for $<\sigma V>$ which is calculated by eq.4.

In this paper, the following model for $<\sigma V>$ is proposed in order to decrease the above mentioned problems [6].

$$<\sigma V>_{ab}=C_1X^{-\frac{5}{6}}Y^2e^{\left(-3YX^{\frac{1}{3}}\right)}+C_8(KT)^{-\frac{3}{2}}e^{-\frac{148}{KT}} \qquad (5)$$

Where

$$X=1-\frac{C_2KT+C_4(KT)^2+C_6(KT)^3}{1+C_3KT+C_5(KT)^2+C_7(KT)3} \qquad (6)$$

$$Y=\frac{C_0}{(KT)^{\frac{1}{3}}} \qquad (7)$$

Now we calculate sigmavi for the following important reactions [2].

$$D+{}^3He\rightarrow\alpha+p+18.35\ MeV \quad (8)$$

$$p+{}^{11}B\rightarrow 3\alpha+8.68\ MeV \qquad (9)$$

In the reactions above, **α** represent $\left({}^4_2He\right)$and **D** represent ^2H. **p**is proton. In the model suggested for $<\sigma V>$, constant coefficients C_0 toC_8 are calculated by fitting the eq.5 with available data [7] for each of the above reactions. The results has shown in table (1)and we have obtained the function of Sigmavi in respect to KT.The graph of Sigmavi with respect to KT is drawn in figures 1and 2 for each of the above reactions.

Calculating Lawson criterion for each of the discussed reactions:
At first, for the arbitrary reaction of eq.1 the relations are written. Then substituting desired cores with **a** and **b** according to what was said in introduction, if the output and input energy are E_{out} and E_{in} respectively, the relation $E_{out}>E_{in}$ must be correct during a determined time τ . τ May be the time needed for a complete cycle in pulse or periodic fusion system that the system is expected to have steady state [1,2].

The classic theorem of gas kinetics can be used as a basis for the study of fusion environment. Consequently, for the case of general atomic density N with atomic number Z_i ,we have complete ionization product N_i ions per unit volume , and$N_e = N_i Z_i$ electron per unit volume, so that $N = N_i + N_e = N_i + N_i Z_i$, because the cores are light, it can be approximately statedthat $N_i \cong N_e$. In a fusion system, energy can be considered in three different ways. First, energy released from fusion reaction that we shown by E_{fu}. Second all the energies lost via radiation (E_{ra}), and finally heat energy of particles expressed as E_{th}. In fusion environments 100% efficiency is not possible. So just a fraction of energy is available .Hence we have[1,2]:

$$E_{out} = \eta_{fu} E_{fu} + \eta_{ra} E_{ra} + \eta_{th} E_{th} \quad (10)$$

In which η_s are fraction coefficients for each coordinate of energy. Energy injected to the system (E_{in}) is as follows:

$$\eta_{in} E_{in} = E_{ra} + E_{th} \quad (11)$$

If we assume, for simplicity, that fraction coefficients are equal ($\eta_{fu} = \eta_{ra} = \eta_{th} = \eta_{out}$) and system works in a state.That can produce a constant a power in a determined time τ_E , we have [1,2]:

$$E_{fu} = \tau_E P_{fu}, E_{ra} = \tau_E E_{ra}, E_{th} = \frac{3}{2}(N_i + N_e)kT \quad (12)$$

The lost energy by radiation is because of Bermesstrahlung radiations, we have [1,2]:

$$E_{ra} = E_{br} = \tau_E P_{br} = \tau_E A_{br} (kT)^{\frac{1}{2}} \quad (13)$$

In which A_{br} is Bermesstrahlung radiations power constant and equals $A_{br} = 8.25 \times 10^{-30} J^{\frac{1}{2}} \frac{m^3}{s}$, Then the Lawson criterion is written as [1,2,8]:

$$N_i \tau_E > \frac{3kT(1 - \eta_{in}\eta_{out})}{\eta_{in}\eta_{out}\left(\frac{1}{4} Q_{ab} < \sigma v >_{ab} + A_{br}(kT)^{\frac{1}{2}}\right) - A_{br}(kT)^{\frac{1}{2}}} \quad (14)$$

For the discussed reactions with assumption $\eta_{in} = \eta_{out} = \frac{1}{3}$ Lawson criterion is calculated. The results has shown in table (1). The graph of $N_i \tau_E$ with respect to KThas drawn in figures 3 and 4.

Table (1): Values of C_0 to C_8 and $N_i \tau_E$ forreactions$^3He(D,p)\alpha$ and $^{11}B(p,\alpha)2\alpha$.

Reaction	$^3He(d,p)\alpha$	$^{11}B(p,\alpha)2\alpha$
$C_0 (KeV)^{\frac{1}{3}}$	10.272	17.201
$C_1 \left(\frac{m^3}{s}\right)$	1.51×10^{-22}	62.02×10^{-22}
$C_2 (KeV)^{-1}$	6.2192×10^{-3}	-60.057×10^{-3}

$C_3\left(KeV\right)^{-1}$	-2.0280×10^{-3}	202.05×10^{-3}
$C_4\left(KeV\right)^{-2}$	-0.019022×10^{-3}	1.00404×10^{-3}
$C_5\left(KeV\right)^{-2}$	0.13567×10^{-3}	2.0621×10^{-3}
$C_6\left(KeV\right)^{-3}$	0	0.0091653×10^{-3}
$C_7\left(KeV\right)^{-3}$	0	0.00098305×10^{-3}
$C_8\left(\frac{m^3}{s}\right)$	0	5.01×10^{-15}
$Error$	2%	1%
$N_i\tau_E(\frac{s}{m^3})$	3.04×10^{21}	6.72×10^{17}

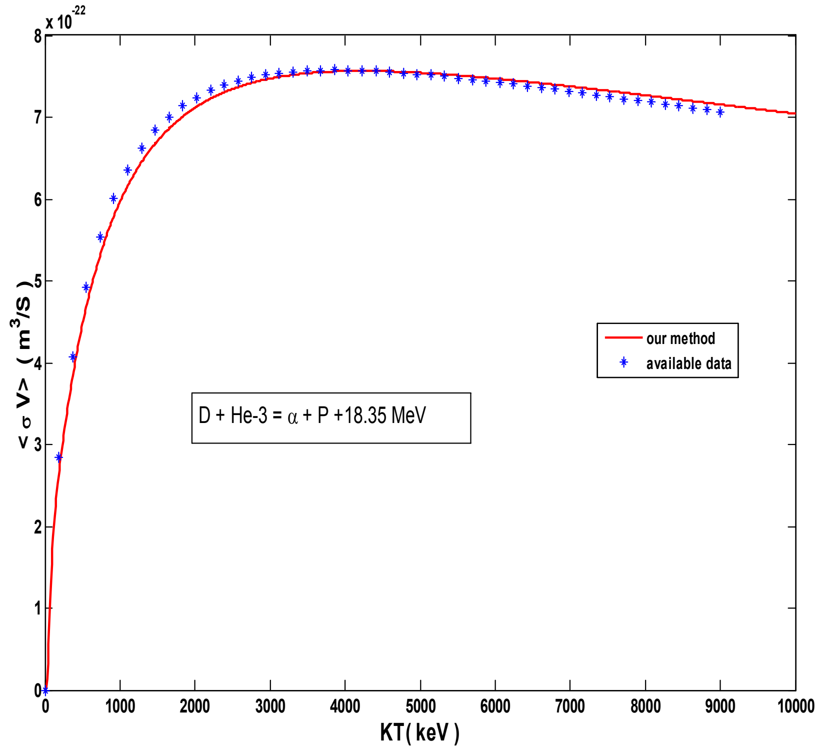

Fig.1 Sigmavi graph in terms of KT for reaction $^3He(D,p)\alpha$.

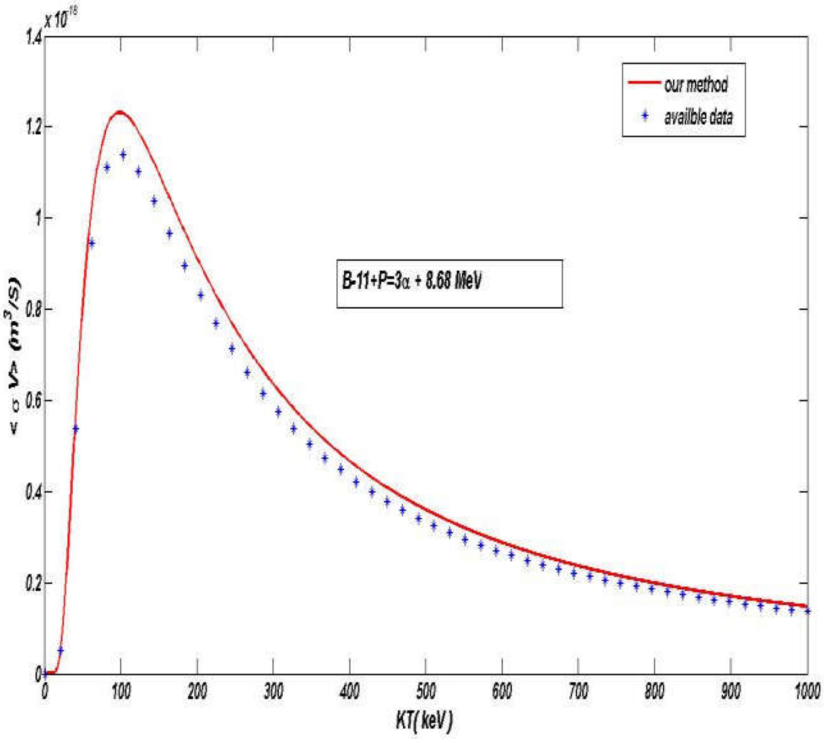

Fig.2 Sigmavi graph in terms of KT for reaction $^{11}B(p,\alpha)2\alpha$.

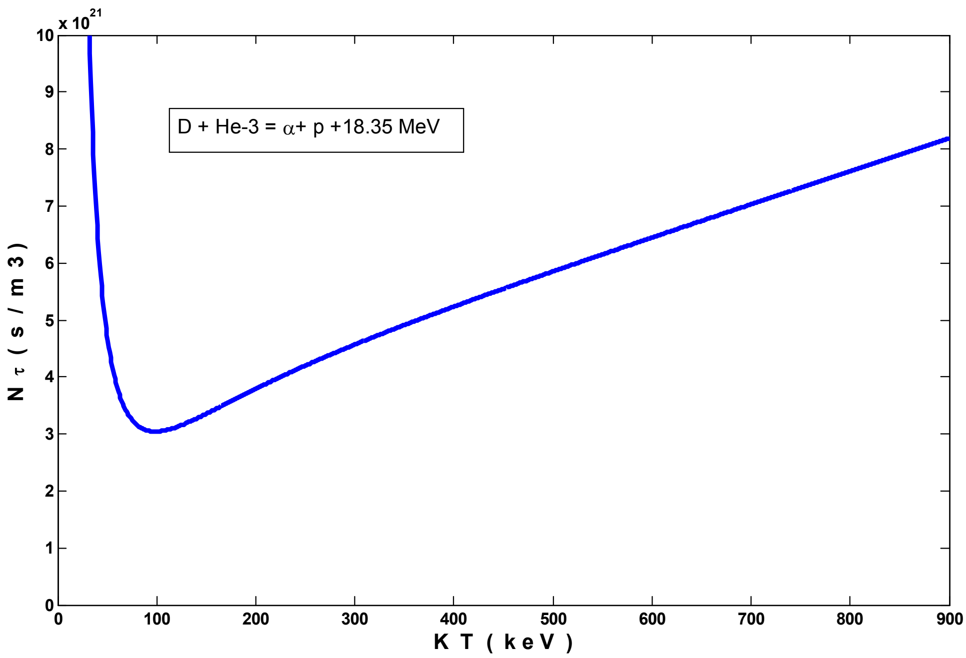

Fig3 Lawson's criterion bounds for reaction $^{3}He(D,p)\alpha$.

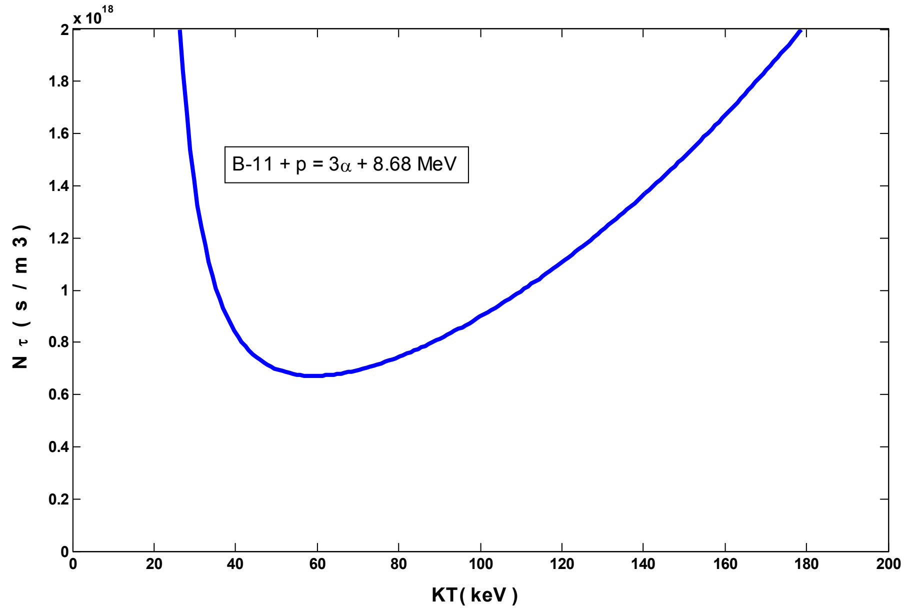

Fig.4 Lawson's criterion bounds for reaction $^{11}B(p,\alpha)2\alpha$.

CONCLUSION

The suggested model for Sigmavi has acceptable answers in low energy. The values calculated for Sigmavi are equal to those of different references. Figurese3 and 4 illustrate very well that $N_i\tau_E$ reaches its lowest value in a specific temperature. The least value of $N_i\tau_E$ is Lawson criterion that we have obtained .This result is very useful because it defines the necessary condition between particles density and energy confining time τ_E and the temperature of the system. But the disadvantage of this system is that all the parameters are not considered. For example, adjusting some parameters to constant values during time τ_E may not be possible. However, this equation has many practical applications.

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