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Reliability and MTTF Analysis of Generator Plant in Industry by Boolean Function

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ABSTRACT
This paper deals with evaluation of reliability and MTTF of a complex system with the aid of Boolean function expansion algorithm technique. The system consists of generators, cables, switches and jumper. Each component of the system has two states, viz, good or bad (failed). The object of the system is to supply power from a distant power house to several section of a rubber factory through cables and switches. The failure times of all components in the complex system follow Weibull/exponential distribution. At the end, numerical computation and some graphs have also been given so as to select which type of system is best suited to perform the mission.
***Keywords**. Boolean function, failure rates, MTTF, Reliability*

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INTRODUCTION

The virial coefficients provide valuable information about the intermolecular forces. Using different type In various reliability systems, quite a number of studies have been carried out by earlier researchers to evaluate reliability expressions for various complex systems by the Boolean function technique. Some auther's have worked on Boolean function. i. e. Fratta et. al. [3] have studied on a Boolean function for computing the terminal reliability in a communication net work, Nakagawa [4] has studied on a decomposition method for computing system reliability by Boolean function expression. Narmada and Jacob [5] have worked on Reliability analysis of a complex system with a deteriorating standby unit under common-cause failure and critical human error. Jacob and Narmada [6] have discussed on Analysis of a two unit deteriorating standby system with repair. Kent [7] has studied on Reliability model and Assessment of the start-graph Network. Recently, Narmada [8] has discussed on Reliability evaluation of missing and boiling power plant in a rubber factory with the aid of bf expansion algorithm technique. Today, there is a vast number of problems which are still solved only on the basis of logical reasoning and experience and not with the aid of reliability calculation. Very little work has been done, so far, in evaluating the reliability of a complex system by Boolean function expansion algorithm technique. Keeping these facts in view, the author in this paper have therefore evaluated the reliability and MTTF of a power plant by using Boolean function expansion algorithm technique.

In this paper, the authors have however considered the power plant consisting of a power house connected with two generators G_1 and G_2 in parallel redundancy. These generators are further connected through 2 cables, three switches and a jumper. The movie of the complex system is to supply power generated by generators to several critical destinations through cables and switches as shown in Fig 1. Here it is assumed that the failure rates for various components of the complex system follow arbitrary distribution. Here it is assumed that the failure rates for various components of the complex system follow arbitrary distribution. It is assumed that no service facility is there to repair the power plant. Reliability of the complex system has been computed when the failure rate of component follow either Weibull or exponential distributors. Moreover, MTTF an important parameter of reliability is also computed for exponential failure rates of the units comprising the system. A numerical example alongwith graphs has also been appended at the end so as to fore cast the operational behaviors of the system at any time.

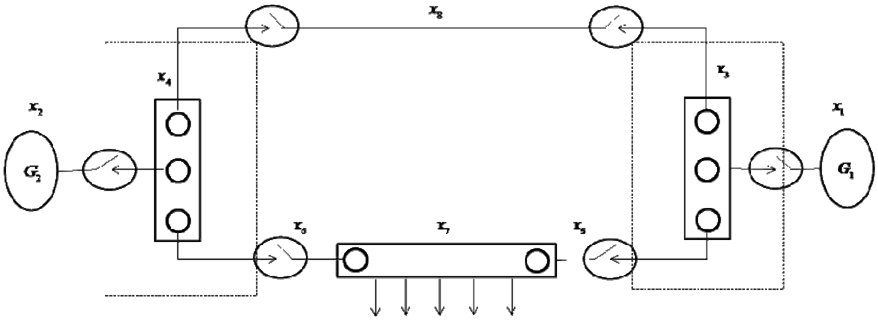

Fig. – 1: System Configuration

ASSUMPTION

1. Initially all components of the system are good.
2. The complex system consists of two generators, three switches, two cables and a jumper.
3. The reliability of all constituent components of the systems are known in advance.
4. The states of all components are statistically independent.
5. In the system, the state of only one component can change at any instant.
6. Failure rates of all the components are arbitrary.
7. The system can fail only when both the generators fail or at least one component is defective.
8. There is no repair facility.

NOTATION

x_1, x_2	states of the generators (G_1, G_2)
x_3, x_4	states of main switch board.
x_7	state of switch board
x_5, x_6	states of cables
x_8	state of jumper
x'_i	negation of x_i for i = 1 – 8
$\wedge$	conjunction
$\vee$	disjunction

$$x_i = \begin{cases} 0, & \text{is the bad state} \\ 1, & \text{is the good state, } (i = 1-8) \end{cases}$$

$P(\phi = 1)$ the probability of the successful operation of the function φ.

FORMATION OF MATHEMATICAL MODEL

By using BF technique, the conditions of capability for the successful operation of the system in terms of logical matrix are expressed as

$$\phi(x_1, x_2, \ldots\ldots\ldots\ldots, x_8) = \begin{bmatrix} x_1 & x_3 & x_5 & x_7 & 0 & 0 \\ x_1 & x_3 & x_8 & x_4 & x_6 & x_7 \\ x_2 & x_4 & x_6 & x_7 & 0 & 0 \\ x_2 & x_4 & x_8 & x_3 & x_5 & x_7 \end{bmatrix} \tag{1}$$

SOLUTION OF THE MODEL

By the application of the algebra of logic, equation (1) may be written as

$$\phi(x_1, x_2, \ldots\ldots x_8) = x_1 \wedge f(x_1, x_2, \ldots\ldots x_8) \tag{2}$$

Where

$$f(x_1, x_2, \ldots\ldots\ldots\ldots, x_8) = \begin{bmatrix} x_1 & x_3 & \vline & x_5 & 0 & 0 \\ 0 & 0 & \vline & x_8 & x_4 & x_6 \\ x_2 & x_4 & \vline & x_6 & 0 & 0 \\ 0 & 0 & \vline & x_8 & x_3 & x_5 \end{bmatrix} \tag{3}$$

In equation (3) five arguments are entered twice. Let us take x8 and break up the complete event f into incompatible events as follows.

$$f(x_1, x_2, \ldots\ldots x_8) = x'_8 y_0 \vee x_8 y_1 \tag{4}$$

Where

$$y_0 = \begin{bmatrix} x_1 & x_3 & x_5 \\ x_2 & x_4 & x_6 \end{bmatrix} \tag{5}$$

$$y_1 = \begin{bmatrix} x_1 & x_3 & \vline & x_3 & 0 \\ 0 & 0 & \vline & x_4 & x_6 \\ x_2 & x_4 & \vline & x_6 & 0 \\ 0 & 0 & \vline & x_3 & x_5 \end{bmatrix} \tag{6}$$

All the arguments enter in equation (5) only once and hence y0 is non-iterated. Also arguments (x_1, x_4, x_5, x_6) still enter twice in y1. Let us expand the function y1 by argument x_3 as follows.

$$y_1 = x'_3 y_{10} \vee x_3 y_{11} \tag{7}$$

Where

$$y_{10} = \begin{bmatrix} x_2 & x_4 & x_6 \end{bmatrix} \tag{8}$$

$$y_{11}=\begin{bmatrix} x_1 & 0 & x_5 & 0 \\ 0 & 0 & x_4 & x_6 \\ x_2 & x_4 & x_6 & 0 \\ 0 & 0 & x_5 & 0 \end{bmatrix} \tag{9}$$

Function y_{10} is non-iterated. Now we expand function y_{11} by argument x_4.

$$y_{11}=x_3' y_{110} \vee x_3 y_{111} \tag{10}$$

Where

$$y_{110}=\begin{bmatrix} x_1 & x_5 \end{bmatrix} \tag{11}$$

$$y_{111}=\begin{bmatrix} x_1 & x_5 \\ x_2 & x_6 \end{bmatrix} \tag{12}$$

Thus all the functions are now non-iterated and are not subjected to further transformations so making use of equation (4) – (12) we get.

$$f(x_1,x_2,\ldots\ldots\ldots\ldots.,x_8)=\begin{bmatrix} x_8' & x_1 & x_3 & x_5 & 0 \\ 0 & x_2 & x_4 & x_6 & 0 \\ x_8 & x_3' & x_2 & x_4 & x_5 \\ 0 & x_3 & x_4' & x_1 & x_5 \\ 0 & 0 & x_4 & x_1 & x_5 \\ 0 & 0 & 0 & x_3 & x_6 \end{bmatrix} \tag{13}$$

Equation (13) can be written as

$$f(x_1,x_2,\ldots\ldots\ldots\ldots.,x_8)=\begin{bmatrix} C_1 & x_1 & x_3 & x_5 \\ 0 & x_2 & x_4 & x_6 \\ C_2 & x_2 & x_4 & x_6 \\ C_3 & x_1 & x_5 & 0 \\ C_4 & x_1 & x_5 & 0 \\ 0 & x_2 & x_6 & 0 \end{bmatrix} \tag{14}$$

Where $C_1=x_8'$, $C_2=x_8x_3'$, $C_1=x_8x_3x_3'$, $C_1=x_8x_3x_4$

Now, the probability of successful operation of the function f is given by.

$$P(f=1)=P(C_1)P(f|C_1)+P(C_2)P(f|C_2)+P(C_3)P(f|C_3)+P(C_4)P(f|C_4) \tag{15}$$

$$P(f=1)=P(x_8')P(y_0)+P(x_8x_3')P(y_{10})+P(x_8x_3x_4')P(y_{110})+P(x_8x_3x_4)P(y_{111}) \tag{16}$$

Where the events C_1 (i = 1 - 4) form a complete group of incompatible hypotheses and P(f | C_1) form the conditional probabilities of a good state of the system for each hypothesis. If R_1 is the reliability of the component of the complex system corresponding to state x_1 (i = 8) and Qi is the corresponding unreliability. Where $S_i = 1 - R_i$ then we get.

$$P(x_8')=S_8$$

$$P(y_0)=1-(1-R_1R_3R_5)(1-R_2R_4R_6)$$

$$P(x_8x_3')=R_8S_3$$

$$P(y_{10})=R_2R_4R_6$$

$$P(x_8x_3x_4')=R_8R_3S_4$$

$$P(y_{110})=R_1R_5$$

$$P(x_8x_3x_4)=R_8R_3R_4$$

$$P(y_{111})=1-(1-S_1S_2)(1-S_5S_6)$$

Finally, the probability of successful operation of the complex system is given by

$$\begin{aligned} R_8 &= P(\phi=1)P(x_7)P(f=1) \\ &= R_7S_8\left[1-(1-R_1R_3R_5)(1-R_2R_4R_6)\right]+R_7R_8S_3(R_2R_4R_6) \\ &\quad +R_3R_4R_7R_8(R_1R_5)+R_3R_4R_7R_8\left[(1-S_1S_2)(1-S_5S_6)\right] \end{aligned} \tag{17}$$

Particulars Cases

Case I: If the reliability of each component of the complex system is R, equation (17) gives

$R_s = 2R^4 + 2R^7 – 5R^7 + 2R^8$ (18)

Case II: When the failure rates follow Weibull time distribution.

$$R_w(t) = 2\exp(-4rt^{\alpha}) + 2\exp(-6rt^{\alpha}) - 5\exp(-6rt^{\alpha}) + 2\exp(-8rt^{\alpha}) \quad (19)$$

Where α is a positive parameter.

Case III: When the failure rates follow exponential distribution.

Exponential distribution is a particular case of Weibul distribution for α = 1 and is very useful in several practical problems. Hence, the reliability of the complex system with this case at any instant t is given by

$$R_E(t) = R_W(t) \text{ at } \alpha = 1 \quad (20)$$

and the corresponding expression for MTTF is given by

$$MTTF = \int_0^{\infty} R_E(t)dt = \frac{0.2467}{r} \quad (21)$$

Numerical computation for reliability

Setting r = 0.02 and α = 1 in equations (19) and (20).

Numerical computation for MTTF

Setting r = 0, 0.1, 0.2, ……1. is equalities (21).

INTERPRETATION OF THE RESULTS

From figure - (1) and (2) shows that the reliability of the complex system at any time t when failure follows either exponential or Weibull distribution, where as Fig. (1) and (2) shows the graph of reliability vs time for exponential and Weibull distribution. It can be seen from graph that reliability of the complex system decrease approximately at a uniform rate when failure rates follow exponential distribution, while it decrease very rapidly when failure rates follow Weibull distribution.

Further, Figure – (3) represent the MTTF with respect to failure rate r, i.e. failure rate of each component of the complex system. Fig. 3 represents the curve the MTTF vs. r. Moreover it reveals that is decreases in the beginning catastrophically, but after some time it decreases at a uniform rate.

Thus, for a given set of values of the failure rates of different components of the complex system, one can forecast at any time the reliability of the power supply and mean time to system failure of such a complex system.

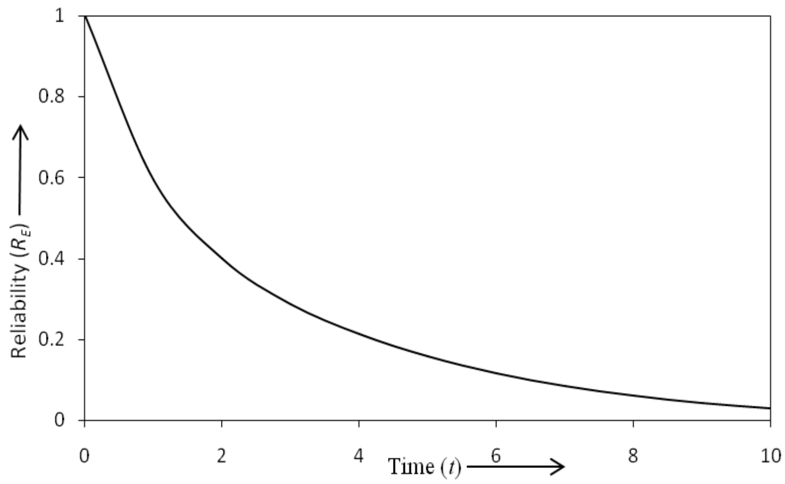

Fig. - 1: Reliability by Exponential distribution Vs Time.

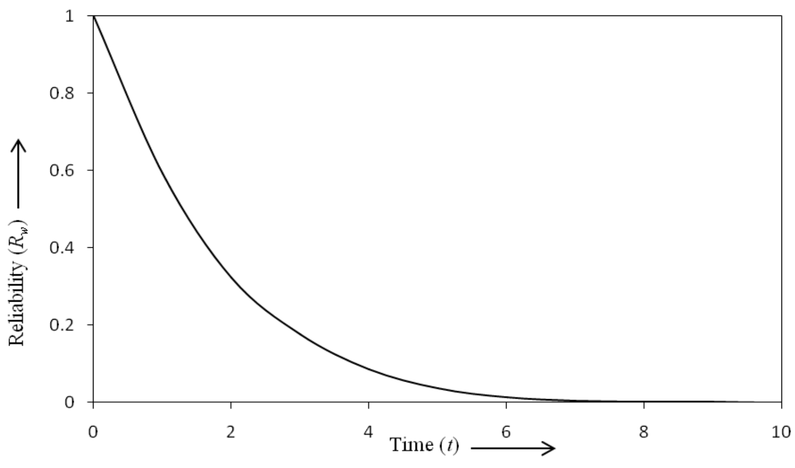

Fig. - 2: Reliability by Weibul distribution Vs Time.

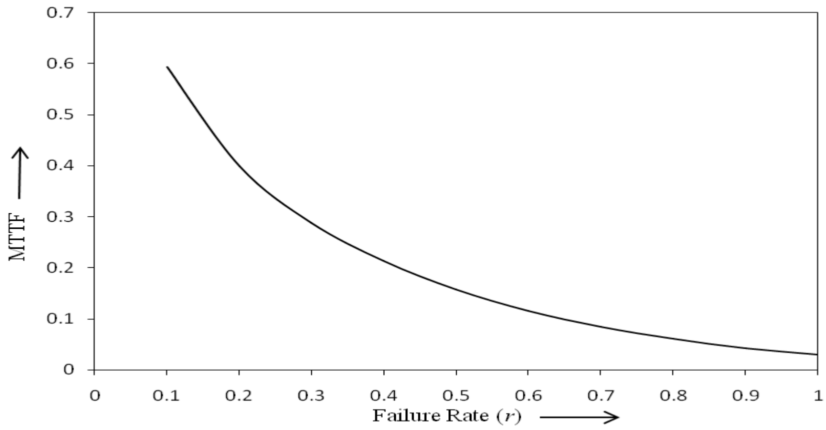

Fig. - 3: MTTF Vs failure rate.

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