

## Heat source effect on Hydro-magnetic convective Heat Transfer through a porous medium in a vertical channel with Radiation effect

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### ABSTRACT

*The present study concentrates an attempt to study an unsteady convective heat and mass transfer of a flow of a viscous electrically conducting fluid through a porous medium confined in a vertical channel bounded by flat walls in the presence of heat generating sources, by taking into account Radiation effect. An oscillatory pressure gradient in the x-direction is considered in the flow region. A perturbation method is employed to solve the equations governing the flow heat and mass transfer has been solved to obtain the expression for velocity, temperature and concentration. The skin friction, the rate of heat and mass transfer has been evaluated for different variations of the governing parameters.*

**Keywords:** Porous medium, vertical channel, Radiation effect, Heat source.

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### INTRODUCTION

Natural convection flows due to the combined buoyancy effects of thermal and species diffusion in a fluid saturated porous medium have many applications, such a geothermal fields, soil pollution, fibrous insulation and nuclear – waste disposal.

The flow of a viscous incompressible fluid bounded by one or two infinite planes with porous walls has gained considerable importance in view of its applications to reduce boundary layer to a turbulent may be suppressed is to reduce mass from the boundary layer through pores or slits on the boundary.

To obtain any desired reduction in the drag increasing suction along is uneconomical as the energy consumption of the suction pump will be more. Therefore the method of “cooling of the wall” in controlling the laminar flow together with applications of suction has become more useful.

In fluid flows through porous media the finite element method has been used extensively and found to be highly successful. Sugunamma[6] has analysed the free convective heat transfer flow of a viscous fluid through a porous medium confined in horizontal channel by using the Galerkin finite element method. In all these studies the thermal diffusion is not considered. This assumption is true only when the flow takes place at low concentration level. There are, however some exceptions. In view of the importance of this diffusion thermo effect Jha and Singh [4], Ajay Kumar Singh [2], and Riah and Hsui [7] have analysed the convective heat and mass transfer with Soret effect under different conditions.

**MATHEMATICAL FORMULATION**

We analyse the convective flow of an incompressible viscous fluid in a horizontal channel filled with a porous matrix bounded by parallel porous walls. The flow takes place along the axis of the channel. The surfaces of the walls are kept at constant heat and mass flux in the direction of the flow. The momentum equations for the fully developed free convection flow. It is assumed that the fluid is in local thermal equilibrium. Boussinesq approximation is used so that the density variation will be retained on the buoyancy term. We choose the Cartesian frame of reference  $O(x,y,z)$ , such that the imposed pressure gradient is along  $x$ -axis and  $y = \pm h$  are the boundary planes (Fig.(a)). In the absence of extraneous forces the flow is unidirectional along  $x$ -axis.

Under these assumptions the equations governing the flow and heat transfer are

$$\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial y^2}\right) - \left(\frac{\mu}{k}\right)v_0 \frac{\partial u}{\partial y} = \frac{1}{\rho} \left(\frac{\partial p}{\partial x}\right) \tag{1.1}$$

$$0 = \left(\frac{\partial p}{\partial y}\right) - \rho g \tag{1.2}$$

$$-v_0 \frac{\partial T}{\partial y} = k_f \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2}\right) + Q(T - T_e) - \left(\frac{\partial(q_r)}{\partial y}\right) \tag{1.3}$$

$$\rho = \rho_0 [1 - \beta(T - T_0)] \tag{1.4}$$

By using Rosseland approximation (Brewster\*) the radiative heat flux is given by

$$q_r = - \left(\frac{4\sigma^*}{3\beta_R} \frac{\partial(T'^4)}{\partial y}\right) \tag{1.5}$$

and expanding  $T'^4$  by Taylor's series and neglecting terms of higher order we get

$$T'^4 \cong 4T_e^3 T - 3T_e^4 \tag{1.6}$$

where  $\sigma^*$  is the Stefan-Boltzman constant and  $\beta_R$  is mean absorption coefficient.

The flow being unidirectional, in view of the equation of continuity  $u = u(y, z)$ .

The heat mass flux being constant along the channel

$$\frac{dT}{dx} = A, \text{ on the wall } y = h$$

Where A is the uniform temperature gradients respectively. Hence the temperature in the flow field may choose to be

$$T = Ax + T_1(y, z)$$

The boundary conditions are

$$\begin{aligned} u &= 0 && \text{on } y = h \\ T &= T_1 && \text{on } y = h \text{ at the entry } x = 0 \text{ and} \\ T &= Ax + T_1 && \text{on } y = h, x \neq 0 \end{aligned} \tag{1.7}$$

In view of the symmetry w.r.t. the central line  $y = 0$

$$\frac{\partial u}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = 0 \quad \text{on } y = 0 \tag{1.8}$$

We introduce the following non-dimensional variables as follows.

$$z = z^*b, \quad y = y^*b, \quad T = T_0 + \theta^* (T_1 - T_0) \quad u^* = \frac{v u}{\beta g b^2 (T_1 - T_0)}$$

Substituting these Non-dimensional variables in equations (1.1)-(1.4)

$$\frac{\partial}{\partial y} \left( \frac{\partial^2 u}{\partial z^2} \right) + \frac{\partial^3 u}{\partial y^3} - D^{-1} + s \left( \frac{\partial^2 u}{\partial y^2} \right) = N_t \quad (1.9)$$

$$\left( 1 + \frac{4}{3N} \right) \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} + ps \frac{\partial \theta}{\partial y} - \alpha \theta = N_t PGU \quad (1.10)$$

Where

$$Nt = \frac{Ab}{T_1 - T_0} \quad (\text{Non-dimensional temperature gradient})$$

$$P = \frac{\mu C_p}{k_1} \quad (\text{Prandtl number}), \alpha = \frac{QL^2}{k_f} \quad (\text{Heat source parameter})$$

$$G = \frac{\beta g b^3 (T_1 - T_0)}{\nu^2} \quad (\text{Grashof number.}), Ec = \frac{\beta g b}{C_p} \quad (\text{Eckert number})$$

$$N = \frac{\beta_r K_f}{4\sigma^* T_e^3} \quad (\text{Radiation parameter})$$

The corresponding boundary conditions in the non-dimensional form are

$$u = 0 \quad \text{on } y = 1 \quad \theta = 1, \quad \text{at } x = 0 \quad \text{on } y = 1 \quad (1.11)$$

$$\theta = 1 + N_1 x, \quad x \neq 0 \quad \text{on } y = 1 \quad (1.12)$$

$$\frac{\partial u}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = 0 \quad \text{on } y = 0 \quad (1.13)$$

In view of the two dimensionality and symmetry of the flow w.r.t. the midplane of the channel we analyse the flow features in a domain in the upper half of the channel bounded by the impermeable wall lying between two parallel planes normal to the wall at unit distance apart. The finite element analysis with quadratic approximation functions is carried out using eight noded serendipity elements.

## RESULTS AND DISCUSSION

The Fintie Element analysis is used to investigate the mixed convective Heat Transfer flow of a viscous incompressible fluid through a saturated porous medium in a Horizontal channel through a flow with in the porous medium is taken into account based on the Brinkaman extended Darcy model the flow is uni directional and the behaviour of velocity temperature and Nusselt number is discussed for different variations of the governing parameters. The variation of u at different vertical levels Z=0 and 1 are drawn in Figs.1 and 2. these profiles are asymmetric curves exhibiting a reversal flow in the region 0≤y≤0.6 at Z=0 and Z=1. Lesser the permeability of the porous medium larger u at Z=0 level reversal flow in the vicinity of y=1. Lesser the permeability of the porous medium larger u at Z=0 level Larger u in the flow region and for further lowering of the permeability smaller the velocity in the flow region (Fig.1). Fig.2 shows that at Z=1 level a reversed behaviour is noticed in u with D<sup>-1</sup>. Also the reversal zone shrinks with D<sup>-1</sup>≤5×10<sup>3</sup> and enhances with D<sup>-1</sup>≥10<sup>4</sup>.

The non-dimensional temperature θ is shown in Figs.3 and 4 at different horizontal and vertical levels. We follow the convention that the non-dimensional temperature positive or negative according as the actual temperature is greater/lesser than in mean temperature. We notice that the non-dimensional temperature is negative for all variations this indicates that actual temperature is lesser than the mean temperature every where in the fluid region lesser the porous permeability smaller the actual temperature in the every flow region at all vertical and horizontal levels (Figs.3 and 4)

The shear stress has been evaluated for variations in D<sup>-1</sup> and S. At three different axial positions Z=0, and 1 of the channels and is exhibited in Tables.1 and 2. It is observed from these tables that lesser the permeability of the porous medium larger the shear stress at all axial positions (Tables.1 and 2). The Nusslet Number (Nu) which represents the rate of Heat Transfer has been evaluated for variations in G, D<sup>-1</sup>, and S. The Nusselt Number at two

different position at  $Z=0$  and  $1$  across the boundary plane has been shown in Tables.3 and 4 for different variations. The analysis has been carried out with Prandtl Number  $P=0.71$ . It is noticed that the Nusselt Number ( $Nu$ ) decreases as we move across the boundary plane at all axial positions where as its magnitude decreases with increase in the axial distance starting from the entry position  $Z=0$ . Fixing the other parameters, in general the rate of Heat Transfer enhances with  $D^{-1}$  and  $|G|$  at all axial distances. An increasing the suction parameter  $S \leq 0.3$  enhances  $|Nu|$  and depreciates with higher  $S \geq 0.3$  (Tables.3 and 4).

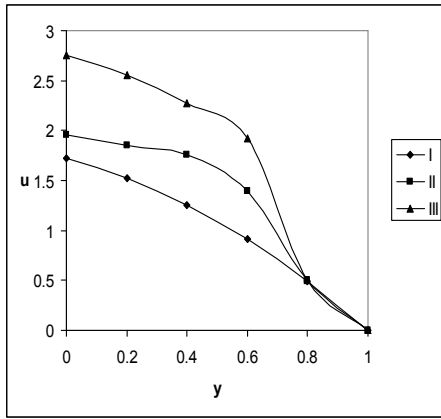


Fig. 1 : u with  $D^{-1}$  at  $z = 0$

I      II      III  
 $D^{-1}$   $10^3$   $5 \times 10^3$   $10^4$

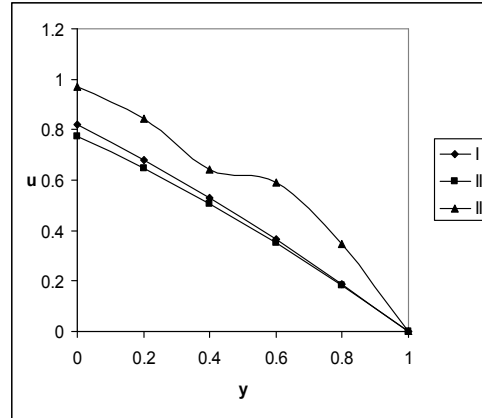


Fig. 2 : u with  $D^{-1}$  at  $z = 0$

I      II      III  
 $D^{-1}$   $10^3$   $5 \times 10^3$   $10^4$

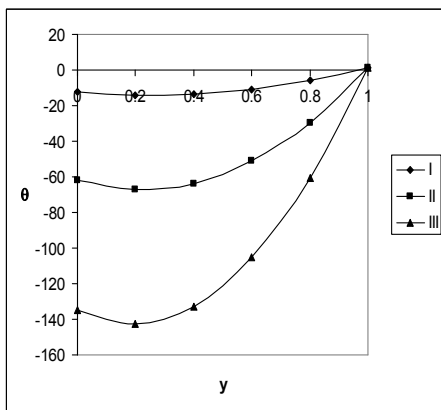


Fig. 3 :  $\theta$  with  $D^{-1}$  at  $z = 0$

I      II      III  
 $D^{-1}$   $10^3$   $5 \times 10^3$   $10^4$

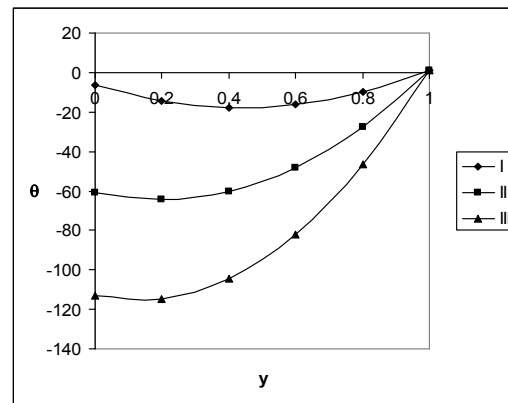


Fig. 4 :  $\theta$  with  $D^{-1}$  at  $z = 0$

I      II      III  
 $D^{-1}$   $10^3$   $5 \times 10^3$   $10^4$

Table -1 Stress ( $\tau$ ) at  $Z=0$

$D^{-1}$	I	II	III
$10^2$	77.0146	76.0663	75.1302
$3 \times 10^2$	231.335	228.973	226.64
$5 \times 10^2$	386.013	382.283	378.599

Table -2 Stress ( $\tau$ ) at  $Z=1$

$D^{-1}$	I	II	III
$10^2$	55.8535	55.1435	54.4468
$3 \times 10^2$	131.845	130.276	128.733
$5 \times 10^2$	207.136	204.711	202.325

S	0.1	0.3	0.5

**Table -3 Nusselt Number (Nu) at Z=0**

D <sup>-1</sup>	I	II	III	IV	V	VI	VII	VIII
10 <sup>2</sup>	16.962	46.9883	77.0146	-13.0642	-43.0905	-73.1168	150.19	75.1302
3x10 <sup>2</sup>	47.826	139.58	231.335	-43.9282	-135.683	-227.437	456.002	226.64
5x10 <sup>2</sup>	78.761	232.389	386.013	-74.8639	-228.489	-382.115	762.622	378.599

**pTable - 4 Nusselt Number (Nu) at Z=1**

D <sup>-1</sup>	I	II	III	IV	V	VI	VII	VIII
10 <sup>2</sup>	12.7298	34.2917	55.8535	-8.83203	-30.3939	-51.9557	108.344	54.4468
3x10 <sup>2</sup>	27.9282	79.8868	131.845	-24.0304	-75.989	-127.948	258.61	128.733
5x10 <sup>2</sup>	42.9864	125.061	207.136	-39.0886	-121.164	-203.238	407.479	202.325

G	10	30	50	-10	-30	-50	100	100
S	0.1	0.1	0.1	0.1	0.1	0.1	0.3	0.5

**Nomenclature**

- U velocity(m/s)
- p pressure(N/m<sup>2</sup>)
- T temperature in the flow region(K)
- K<sub>f</sub> The coefficient of thermal conductivity(W/mk)
- Q is the strength of the heat source
- T<sub>0</sub> Mean Temperature
- q<sub>r</sub> Radiative heat flux

**Greek Symbols**

- ρ<sub>0</sub> Mean density
- ρ Density (Kg/m<sup>3</sup>)
- μ Coefficient of viscosity(Ns/m<sup>2</sup>)
- β The coefficient of thermal expansion(W/mk)

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