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Trapezoidal Hesitant Fuzzy Multi- Attribute Decision Making Based on TOPSIS

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ABSTRACT
In this paper, the problems faced by the farmers with regard to oil seeds cultivation in Villupuram district are being discussed using fuzzy decision making tools. Through this paper, TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method is being utilized for the newly introduced Trapezoidal Hesitant Fuzzy Set (TrHFS) to solve the uncertainty in the decision making process.
Keywords: *Multi-Attribute Decision Making, Hesitant Fuzzy Set, Trapezoidal Hesitant Fuzzy Set, Technique for Order Preference by Similarity to Ideal Solution, TOPSIS.*

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INTRODUCTION

Vagueness and uncertainty in the real world-Multi Criteria Decision Making problems make the situation very complex. This complexity and impreciseness pave way for inaccuracy into the criteria's weights and evaluation values of alternatives and in turn make it for researchers very hard to quantify it. To reduce the vagueness, A.L Zadeh [18] introduced Fuzzy Sets in the year 1965. Fuzzy Sets has been emerged as one of the foremost theory which enumerates the qualitative data in the large scale. However, in Fuzzy Sets the membership degree of the element is represented by a single value between zero and one, and a major setback of Fuzzy Sets is that often single values do not convey information correctly. In such cases, the concept of Fuzzy Number has been introduced to quantify the qualitative arguments.
In some cases, the membership degree of an element is not a single value but a set of values. Such situations are managed by Hesitant Fuzzy Sets (HFSs). Hesitant Fuzzy Sets are first introduced by Torra [11] and it permits the membership degree of an element to be a set of several values between 0 and 1. Wang et al provided an outstanding approach with HFSs to solve MCDM problems. After the pioneering work of Torra, the HFS has received much attention and has been used in decision making and clustering analysis. Xia and Xu [16, 17 and 21] studied the aggregation operators of Hesitant Fuzzy Sets and applied them to decision making.
In this paper, an extension has been made and developed by fitting the Trapezoidal Hesitant Fuzzy Set (TRHFS) approach with decision making package TOPSIS. The problems faced by the farmers in Villupuram District are analyzed through our newly designed Trapezoidal Hesitant Fuzzy Multi-Attribute Decision Making (TRHF - MADM) method.
This paper has been organized in the following manner. The concepts of HFS, TrHFS and some of its basic properties have been introduced in section 2. Section 3 presents the working algorithm for TOPSIS. In Section 4 an application which uses hesitant information is presented. Section 5 deals with the adaptation and description of the problem and finally the paper is concluded in Section 6 with the interpretation of the final results.

BASIC DEFINITIONS

2.1 Fuzzy Set

Let E be the universal set, let x be an element of E, then the fuzzy subset $\underset{\sim}{A}$ of E, ($\underset{\sim}{A}$ of E) is a set of ordered pairs

$$\underset{\sim}{A} = \{(x \mid \mu_{\underset{\sim}{A}}(x))\}, \text{for all } x \in E \tag{2.1}$$

where $\mu_{\underset{\sim}{A}}(x)$ is the grade (or) degree of membership of x in $\underset{\sim}{A}$.

$\mu_{\underset{\sim}{A}}(x)$ takes the value from the membership set M = [0,1] and

$\mu_{\underset{\sim}{A}}(x)$ is the membership function or characteristic function.

2.2 Hesitant Fuzzy Set

Let X be a fixed set, a Hesitant Fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0,1].

Mathematical representation of Hesitant fuzzy set [18]:

$$A = \{< x, h_A(x) > / x \in X\} \tag{2.2}$$

where $h_A(x)$ is a set of some values in [0,1], denoting the possible membership degrees of the element $x \in X$ to the set *A.*

2.3 Hesitant Fuzzy Element (HFE)

Every $h = h_A(x)$ is defined as hesitant fuzzy element (HFE).

Example Let $X = \{x_1, x_2, x_3, x_4\}$ be a fixed set,

$h_A(x_1) = \{0.2,0.4,0.5,0.4\}$,

$h_A(x_2) = \{0.3,0.4,0.5\}$ and

$h_A(x_3) = \{0.3,0.2,0.5,0.6\}$

$h_A(x_4) = \{0.2,0.4,0.6\}$

be the HFEs of x_i ($i=1,2,3,4$) to the set *A* respectively. Then *A* can be considered as a HFS:

$$A=\{< x_1, \{0.2,0.4,0.5,0.4\}>, < x_2, \{0.3,0.4,0.5\}>, < x_3, \{0.3,0.2,0.5,0.6\}>, < x_4, \{0.2,0.4,0.6\} >\}$$

2.4 Special properties of HFS

(i) Empty Hesitant Fuzzy Set: $h = \{0\}$

(ii) Full Hesitant Fuzzy Set: $h = \{1\}$, denoted as I^*.

(iii) Complete Ignorance: (All are possible) $h = [0,1] \triangleq U^*$

(iv) Nonsense Set: $h = \emptyset^*$

2.5 Score of an HFS

For a Hesitant Fuzzy Element (HFE)*h,*

$$s(h) = \frac{\sum_{\gamma \in h} \gamma}{l_h} \tag{2.3}$$

is called the score of h, where l_h the number of the elements in *h.*

Case(i): For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then h_1 superior to h_2, denoted by $h_1 \succ h_2$

Case(ii): $s(h_1) = s(h_2)$ then h_1 is indifferent to h_2, denoted $h_1 \sim h_2$

Definition 2.6: Fuzzy Number

A Fuzzy number $\underset{\sim}{A}$ is a fuzzy set on the real line R, which must satisfy the following conditions.

(i) $\mu_{\underset{\sim}{A}}(x)$ is piecewise continuous

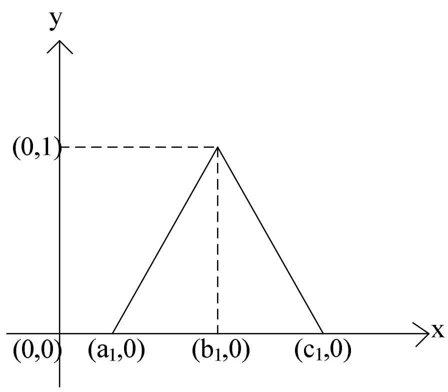
(ii) There exist atleast one $x_o \in R$ with $\mu_{\underset{\sim}{A}}(x_0) = 1$

(iii) $\underset{\sim}{A}$ must be normal and convex

Definition 2.7: Triangular Fuzzy Number

Triangular Fuzzy Number is defined as $\underset{\sim}{A}$ = {a,b,c}, where all a, b, c are real numbers and its membership function is given below.

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} 0 & for\ x < a \\ \frac{(x-a)}{(b-a)} & for\ a \le x \le b \\ 1 & for\ x = b \\ \frac{(c-x)}{(c-b)} & for\ b \le x \le c \\ 0 & for\ x > c \end{cases}$$



2.8 Triangular Hesitant Fuzzy Set

Let X be a fixed set, a Triangular hesitant fuzzy set (THFS) defined by

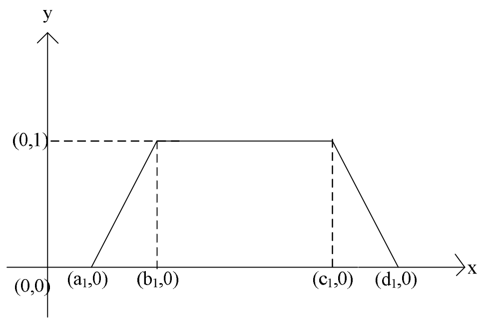
$$A = \{< x, Th_A(x) > / x \in X\} \tag{2.4}$$

where $Th_A(x)$ is a set of some triangular values in (a^L, a^M, a^U).

Definition 2.9: Trapezoidal Fuzzy Number

A fuzzy set $\underset{\sim}{A}$ = (a, b, c, d) is said to trapezoidal fuzzy number if its membership function is given by where a ≤ b ≤ c ≤ d

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} 0 & for\, x < a \\ \dfrac{(x-a)}{(b-a)} & for\, a \le x \le b \\ 1 & for\, b \le x \le c \\ \dfrac{(d-x)}{(d-c)} & for\, c \le x \le d \\ 0 & for\, x > d \end{cases}$$



2.10 Trapezoidal Hesitant Fuzzy Set (TrHFS)

Let X be a fixed set, then a Trapezoidal Hesitant Fuzzy Set (T_rHFS) D on X is described as:

$$D = \{< x, T_r h_A(x) > x \in X\} \tag{2.5}$$

in which $T_rh_A(x)$ is function of $T_r h_A'(x)$ where $T_rh_A'(x) = (a^L, a^{M_1}, a^{M_2}, a^U)$

2.11Trapezoidal Hesitant Fuzzy Element (T_rHFE)

The quadruple $(a^L, a^{M_1}, a^{M_2}, a^U)$ is called Trapezoidal fuzzy element

2.12 Hesitant Multiplicative Aggregation

To quantify the natural statements presented by the decision maker, we employed Saaty's 1-9 scale with its respective meaning.

Table: The comparison between the 0.1-0.9 scale and the 1-9 scale

1-9 scale	0.1-0.9 scale	Meaning
1/9	0.1	Extremely not preferred
1/7	0.2	Very strongly not preferred
1/5	0.3	Strongly not preferred
1/3	0.4	Moderately not preferred
1	0.5	Equally preferred
3	0.6	Moderately not preferred
5	0.7	Strongly preferred
7	0.8	Very strongly preferred
9	0.9	Extremely preferred
Other values between 1 and 9 (2,4,6,8)	Other values between 0 and 1	Intermediate values used to present compromise

3. Trapezoidal Hesitant Fuzzy Algorithm Based on TOPSIS

This section puts forward a framework for determining the ranking orders for all the alternatives under hesitant fuzzy environment. The approach involves the following steps:

Step 1: We construct attribute decision making problem, let H = $\{h_1, h_2, \ldots, h_p\}$ be a set alternatives, X= $\{x_1, x_2, \ldots, x_m\}$ a set of attributes then the Trapezoidal hesitant decision matrix

Step 2: Choose weight vector $W=\{w_1, w_2, \ldots, w_m\}^T$ where $w_i \in [0,1]$ and $\sum_{i=1}^{m} w_i = 1$ for each

attribute according to their importance over the problem.

$$H = \begin{bmatrix} \{trh_{11}\} & \{trh_{12}\} & \{trh_{13}\} & & \\ \{trh_{21}\} & \{trh_{22}\} & \{trh_{23}\} & \cdots & \{trh_{nj}\} \\ \{trh_{31}\} & \{trh_{32}\} & \{trh_{33}\} & & \\ & \vdots & & \ddots & \vdots \\ & \{trh_{mi}\} & & \cdots & \{trh_{mn}\} \end{bmatrix}$$

Step 3: We can calculate the degree of similarity of the positive ideal and negative ideal by the following expression

$$A^+ = \{<x_j, \max_i \{trh_{ij}\} > \mid j = 1,2,\ldots m\} \qquad (3.1)$$

$$A^- = \{<x_j, \min_i \{trh_{ij}\} > \mid j = 1,2,\ldots m\} \qquad (3.2)$$

Step 4: Calculate the separation measures d_i^+ and d_i^- of each alternative A_i from the Trapezoidal hesitant fuzzy positive ideal A^+ and negative ideal A^- respectively by the following expression;

$$d_i^+ = \{\sum_{j=1}^{m} d(h_{ij}, h_j^+) w_j \qquad (3.3)$$

$$d_i^- = \{\sum_{j=1}^{m} d(h_{ij}, h_j^-) w_j \qquad (3.4)$$

Step 5: We find the relative closeness coefficient $c(A_i)$ and corresponding to each alternative A_i to the hesitant fuzzy solution by using the formula**;**

$$c(A_i) = \frac{d_i^-}{d_i^+ + d_i^-} \qquad (3.5)$$

Step 6: Rank the alternatives according to the relative closeness to the hesitant positive ideal and negative ideal solution.

4. Trapezoidal Hesitant Fuzzy Set Application

Pathinathan.T and Johnson Savarimuthu.S [22] studied the problem faced by the farmers who plant cash crops in the Villupuram district. In this paper, we extend our research work by analyzing oil seeds cultivation in the same locality. In Villupuram district, it has been observed that the farmers show a lot of interest in planting oil seeds such as groundnuts, palmlien, sesame, sunflower. Oil seed cultivation depends on water resources from river and tanks. Rivers in Villupuram such as Pennaiyar, Kadilam,Komuki and Varahanathi serve as the major water source of the district and its Vidur, Sathanur, Komuki have stopped functioning they operate only during raining season.

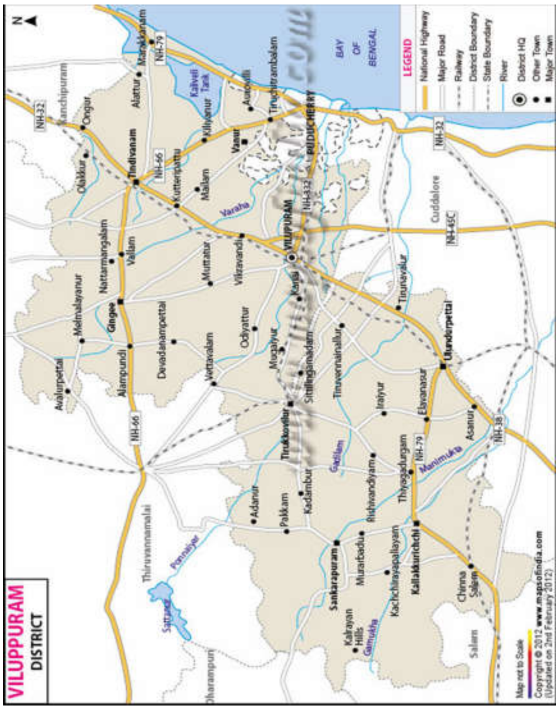


Fig.1

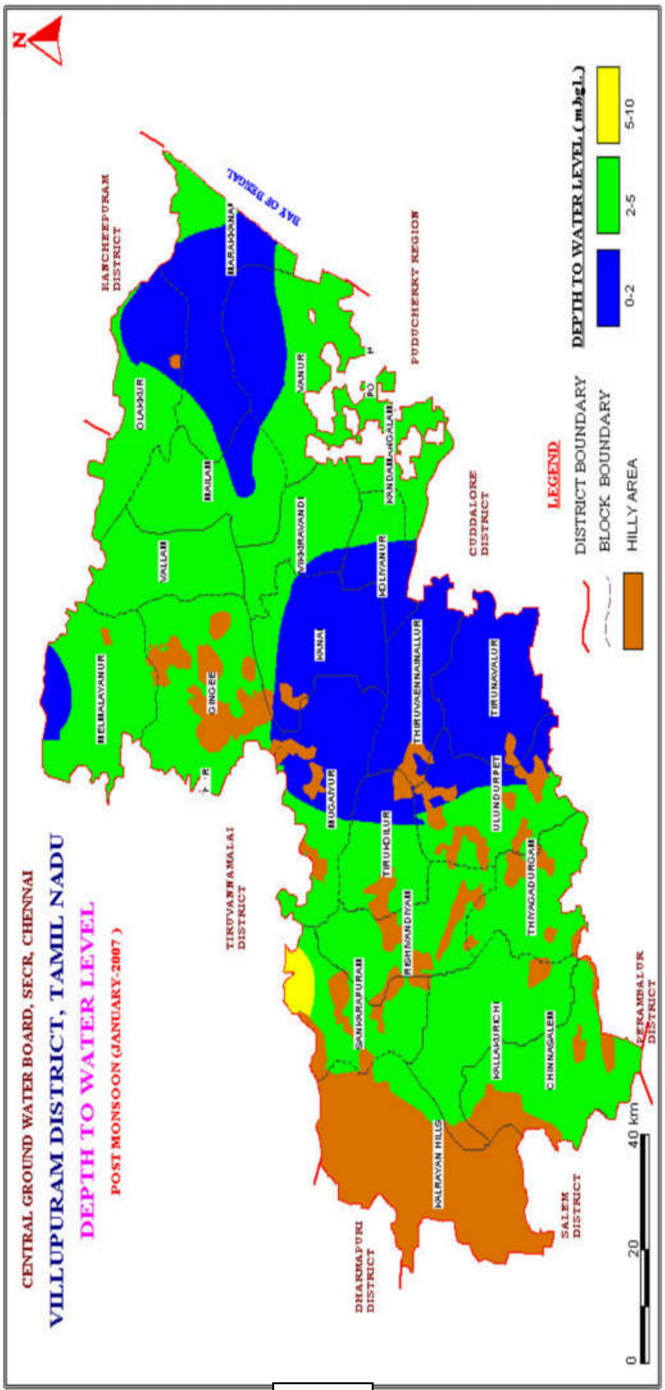


Fig.2

The ground water level is very low due to the failure of the seasonal rains. Due to frequent power cut bore wells cannot be used. Farmers cannot afford to get loan from public banks and private money lending institutions to buy diesel for operating bore wells because it is not possible to pay back the debts.

Faced with these problems, farmers from Thirukovillur, Kalpet and Arasur of Villupuram district have been selling their agricultural land to Real Estate agents for their business.

Interviews were conducted to grasp the struggle of farmers in villupuram district. The present study considers the hierarchical structure of four major cultivating oil crop alternatives with six major attributes which cause severe hindrance in production given away by the experts.

4.1 Experts (from Villupuram District)

We collected the overall information regarding agriculture problems faced in oil seed cultivation from the following Experts in Villupuram District.

DM_1. Mr. S. Kannan (Farmer), Thirukovillur
DM_2. Mr. S. Manikandan (Farmer), Kalpet
DM_3. Mr. Muthuvel, (Farmer), Arasur

4.2 Alternatives

The following are the major oil seeds cultivated in Villupuram District. We took these activities as our alternatives.

A_1. Groundnuts
A_2. Palm lien
A_3. Sesame
A_4. Sunflower

4.3 Attributes

We recorded few major problems that hindered farmers' lives through the interview and consolidated them into six major attributes. These attributes are evaluated into two forms, namely;

(i) Benefit Type (Qualitative in nature)
(ii) Cost Type (Quantitative in nature)

X_1. Crop failure (Benefit Type) – Nutritional need of the people is not met as crop yield has been getting reduced year by year. Crop failure is the result of drying crop and inability to salvage the standing crop due to water scarcity.

X_2. Crop debt (Cost Type) – Money which is borrowed by a farmer such as debt and money borrowed from private money lenders to meet the expenses.

X_3. Lack of water (Benefit Type) – Water scarcity in Sathanur dam and the truant behavior of monsoons.

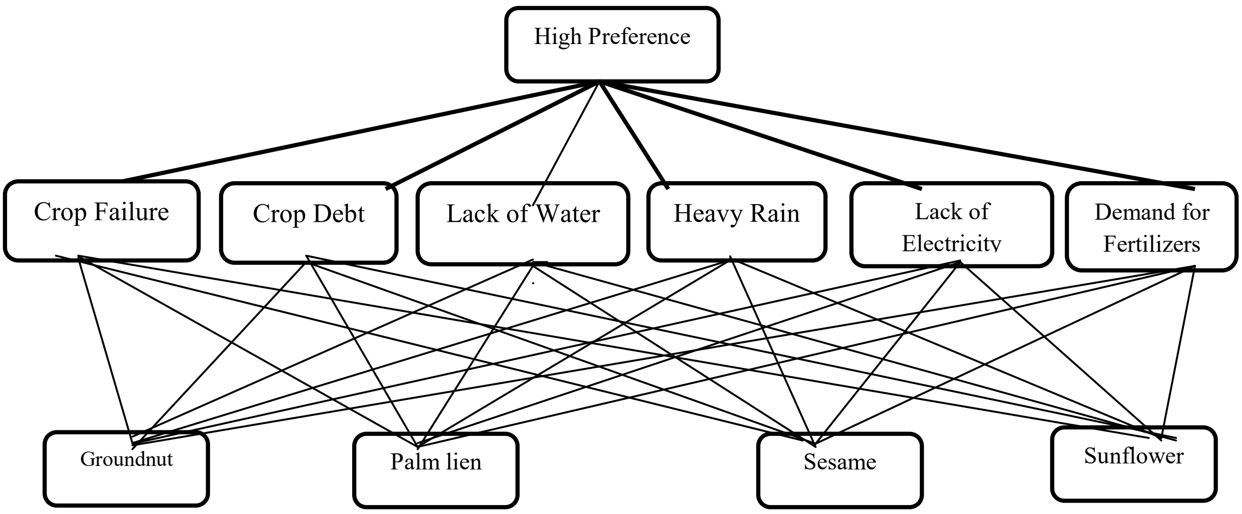
X_4. Heavy rain and Cyclone (*Nilam*) (Benefit Type) – Heavy rain causing soil erosion and soil fertility destroying much livelihood and farmlands in recent years is one of the natural calamities faced by every farmer in the district of Villupuram.

X_5. Lack of Electricity (Cost Type) – Load shedding is one of the reasons for increased use of diesel engines but the cost of diesel is again a burden on the farmers.

X_6. Demand for Fertilizers and Pesticides (Cost Type) – Scarcity of fertilizers and pesticides resulting hatred against the government .

4.4 Hierarchical Structure for TrHFS

The hierarchical structure of this decision making problem is shown from the below diagram;



5. Adaptation and Description of the problem:

Hesitant Fuzzy Decision Matrix is obtained by considering each and every Expert's opinion with their possible membership values and they are recorded as follows:-

Table 5.1: Hesitant Fuzzy Decision Matrix (by utilizing Step 1 in Algorithm)

	X_1	X_2	X_3	X_4	X_5	X_6
A_1	(0.8, 0.4, 0.6)	(0.8, 0.5, 0.6)	(0.6,0.4,0.1)	(0.6,0.4,0.1)	(0.4,-,0.1)	(0.9,-,0.9)
A_2	(0.3,0.5,-)	(0.7,0.8,0.4)	(0.5,0.5,0.1)	(0.6,0.4,0.4)	(0.4,-,0.1)	(0.9,-,0.9)
A_3	(-,-,0.4)	(0.8,0.7,0.5)	(0.5,0.3,0.1)	(0.6,0.4,0.1)	(0.4,-,0.1)	(0.9,-,0.9)
A_4	(0.1,-,0.8)	(0.8,0.7,0.4)	(0.4,0.3,0.1)	(0.6,0.4,0.1)	(0.4,0.7,0.4,)	(0.9,-,0.9)

Where,

A_i , i =1,2,3,4, (four alternatives)

X_i , i =1,2,3,4,5,6 (six attributes) and

$A_1(X_i)$, i=1,2,3,4,5,6 denotes Alternative 1 (groundnuts) comparing with all six attributes. And all the three experts are asked to give their opinions and their opinions are tabulated.

$$A_1(X_i) = (TrM_1, TrM_2, TrM_3,)$$

For instance,

$A_1(X_1)$ = (0.8, 0.4, 0.6,) denotes on discussing Alternative 1 (groundnuts) with attribute 1 (crop failure), Decision Maker 1 (TrM_1) provide 0.8 as the membership value, indicates crop failure considered to be the biggest burden for farmer who indulge in groundnuts cultivation.

Similarly, TrM_2 provide 0.4 as the membership value and so on…

Suppose, if $A_2(X_1)$ = (0.3, 0.5,-), denotes on discussing Alternative 2 (palmlien) with the attribute 1 (crop failure), TrM_3 fail to record his value due to unfamiliarity about the respective alternative over the attribute.

Table 5.2: Hesitant Fuzzy Decision Matrix

	X_1	X_2	X_3	X_4	X	X_6
A_1	(0.8, 0.4, 0.6)	(0.8, 0.5, 0.6)	(0.6,0.4,0.1)	(0.6,0.4,0.1)	(0.4,0.1,0.1)	(0.9,0.9,0.9)
A_2	(0.3,0.5,0.3)	(0.7,0.8,0.4)	(0.5,0.5,0.1)	(0.6,0.4,0.4)	(0.4,0.1,0.1)	(0.9,0.9,0.9)
A_3	(0.4,0.4,0.4)	(0.8,0.7,0.5)	(0.5,0.3,0.1)	(0.6,0.4,0.1)	(0.4,0.1,0.1)	(0.9,0.9,0.9)
A_4	(0.1,0.1,0.8)	(0.8,0.7,0.4)	(0.4,0.3,0.1)	(0.6,0.4,0.1)	(0.4,0.7,0.4,)	(0.9,0.9,0.9)

Table 5.3: Trapezoidal Hesitant Fuzzy Decision Matrix

	X_1	X_2	X_3	X_4	X_5	X_6
A_1	(0.6,0.7,0.8,0.9), (0.2,0.3,0.4,0.5), (0.4,0.5,0.6,0.7)	(0.6,0.7,0.8,0.9), (0.3,0.4,0.5,0.6), (0.4,0.5,0.6,0.7)	(0.4,0.5,0.6,0.7), (0.2,0.3,0.4,0.5), (0.1,0.1,0.2,0.3)	(0.4,0.5,0.6,0.7), (0.2,0.3,0.4,0.5), (0.1,0.1,0.2,0.3)	(0.2,0.3,0.4,0.5), (0.1,0.1,0.2,0.3), (0.1,0.1,0.2,0.3)	(0.9,0.9,0.9,0.9), (0.9,0.9,0.9,0.9), (0.9,0.9,0.9,0.9)
A_2	(0.1,0.2,0.3,0.4), (0.3,0.4,0.5,0.6), (0.1,0.2,0.3,0.4)	(0.5,0.6,0.7,0.8), (0.6,0.7,0.8,0.9), (0.2,0.3,0.4,0.5)	(0.3,0.4,0.5,0.6), (0.3,0.4,0.5,0.6), (0.1,0.1,0.2,0.3)	(0.2,0.3,0.4,0.5), (0.1,0.1,0.2,0.3), (0.1,0.1,0.2,0.3)	(0.2,0.3,0.4,0.5), (0.1,0.1,0.2,0.3), (0.1,0.1,0.2,0.3)	(0.9,0.9,0.9,0.9), (0.9,0.9,0.9,0.9), (0.9,0.9,0.9,0.9)
A_3	(0.2,0.3,0.4,0.5), (0.2,0.3,0.4,0.5), (0.2,0.3,0.4,0.5)	(0.6,0.7,0.8,0.9), (0.5,0.6,0.7,0.8), (0.3,0.4,0.5,0.6)	(0.3,0.4,0.5,0.6), (0.1,0.2,0.3,0.4) (0.1,0.1,0.2,0.3)	(0.4,0.5,0.6,0.7), (0.2,0.3,0.4,0.5), (0.1,0.1,0.2,0.3)	(0.2,0.3,0.4,0.5), (0.1,0.1,0.2,0.3), (0.1,0.1,0.2,0.3)	(0.9,0.9,0.9,0.9), (0.9,0.9,0.9,0.9), (0.9,0.9,0.9,0.9)
A_4	(0.1,0.1,0.2,0.3), (0.1,0.1,0.2,0.3), (0.6,0.7,0.8,0.9)	(0.6,0.7,0.8,0.9), (0.5,0.6,0.7,0.8), (0.2,0.3,0.4,0.5)	(0.2,0.3,0.4,0.5), (0.1,0.2,0.3,0.4), (0.1,0.1,0.2,0.3)	(0.4,0.5,0.6,0.7), (0.2,0.3,0.4,0.5), (0.1,0.1,0.2,0.3)	(0.2,0.3,0.4,0.5), (0.5,0.6,0.7,0.8), (0.2,0.3,0.4,0.5)	(0.9,0.9,0.9,0.9), (0.9,0.9,0.9,0.9), (0.9,0.9,0.9,0.9)

Table 5.4: Positive Ideal Solution (by Eqn: 3.1)

	Positive Ideal Solution		Negative Ideal Solution
A_1^+	$\{< x_1$:(0.6,0.7,0.8,0.9) $>\}$	A_1^-	$\{< x_1$:(0.1,0.2,0.3,0.4)$>\}$
A_2^+	$\{< x_2$: (0.6,0.7,0.8,0.9)$>\}$	A_2^-	$\{< x_2$:(0.2,0.3,0.4,0.5)$>\}$
A_3^+	$\{< x_3$:(0.4,0.5,0.6,0.7) $>\}$	A_3^-	$\{< x_3$:(0.1,0.1,0.2,0.3)$>\}$
A_4^+	$\{< x_4$: (0.4,0.5,0.6,0.7)$>\}$	A_4^-	$\{< x_4$: (0.1,0.1,0.2,0.3)$>\}$
A_5^+	$\{< x_5$(0.5,0.6,0.7,0.8)$>\}$	A_5^-	$\{< x_5$: (0.1,0.1,0.2,0.3)$>\}$
A_6^+	$\{< x_6$:(0.9,0.9,0.9,0.9)$>\}$	A_6^-	$\{< x_6$:(0.9,0.9,0.9,0.9)$>\}$

Then, by utilizing the equation (3.3 and 3.4), we have the following distance values;

Hamming Distance Calculations,

$$d_1^+ = 0.25\sqrt{\frac{|0.2-0.6|^2+|0.3-0.7|^2+|0.4-0.8|^2+|0.5-0.9|^2}{3}}$$
$$+0.20\sqrt{\frac{|0.3-0.6|^2+|0.4-0.7|^2+|0.5-0.8|^2+|0.6-0.9|^2}{3}}$$
$$+0.15\sqrt{\frac{|0.1-0.4|^2+|0.1-0.5|^2+|0.2-0.6|^2+|0.3-0.7|^2}{3}}$$
$$+0.15\sqrt{\frac{|0.1-0.4|^2+|0.1-0.5|^2+|0.2-0.6|^2+|0.3-0.7|^2}{3}}$$
$$+0.15\sqrt{\frac{|0.1-0.5|^2+|0.1-0.6|^2+|0.2-0.7|^2+|0.3-0.8|^2}{3}}$$

$$d_1^+ = 0.3978$$

$$d_1^- = 0.25\sqrt{\frac{|0.6-0.1|^2+|0.7-0.2|^2+|0.8-0.3|^2+|0.9-0.4|^2}{3}}$$
$$+0.20\sqrt{\frac{|0.6-0.2|^2+|0.7-0.3|^2+|08-0.4|^2+|0.9-0.5|^2}{3}}$$
$$+0.15\sqrt{\frac{|0.4-0.1|^2+|0.5-0.1|^2+|0.6-0.2|^2+|0.7-0.3|^2}{3}}$$
$$+0.15\sqrt{\frac{|0.4-0.1|^2+|0.5-0.1|^2+|0.6-0.2|^2+|0.7-0.3|^2}{3}}$$
$$+0.15\sqrt{\frac{|0.2-0.1|^2+|0.3-0.1|^2+|0.4-0.2|^2+|0.5-0.3|^2}{3}}$$

$$d_1^- = 0.3984$$

Table 5.5: Hamming Distance for HFS (by Eqn: 3.3)

d_i^+	P-Distance	d_i^-	N-Distance
d_1^+	0.3978	d_1^-	0.3984
d_2^+	0.4498	d_2^-	0.2606
d_3^+	0.3978	d_3^-	0.2634
d_4^+	0.0441	d_4^-	0.4157

By using equation 3.5, we recorded the closeness values among the alternatives and they are tabulated as follows;

Table 5.6: Closeness for HFS (by Eqn: 3.5)

	d_i^-	$d_i^+ + d_i^-$	$c(A_i)$
$c(A_1)$	0.3984	0.7962	0.5003
$c(A_2)$	0.2606	0.7104	0.3668
$c(A_3)$	0.2634	0.6612	0.3983
$c(A_4)$	0.4157	0.8569	0.4851

From the above table we rank the alternatives A_i (i = 1,2,3,4,) as:-

$$A_1 \succ A_4 \succ A_3 \succ A_2$$

CONCLUSION

By aggregating the opinion collected from the three Decision Makers from the table 5.6, we have the preference ranking order relation as $A_1 \succ A_4 \succ A_3 \succ A_2$, (i.e.,) Alternative A_1 (groundnuts) is dominated by all the other alternatives. Sunflower (A_4) and Sesame (A_3) almost share the same ordering position when compared with the other alternatives. It shows that groundnuts cultivation and Sunflower cultivation are the two crops, which secure good yield, income and livelihood to farmers in terms of the observed attributes.

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